

## **Comparison of scheduling policies for a production system with Part grouping**

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### **1. Introduction**

This paper considers the economic lot scheduling problem, ELSP, which is concerned with lot sizing and scheduling the production of several different items on a single machine. The objective of ELSP is to determine lot sizes and a production schedule such that the sum of inventory holding costs and set-up costs is minimized. The problem will be characterized by the following: no more than one product can be produced at a time on the machine, product production rates are deterministic and constant, product setup costs and times are independent of production order, product demand rates are deterministic and constant, demand must be met in the periods in which occur, inventory costs are directly proportional to inventory levels and production capacity is sufficient to meet total demand. This problem which is known NP-hard has been arduous studied in the literature, appearing diverse heuristics procedures for solve it.

In practical situations is common that some characteristics of the classical ELSP, such as demand, item and product rate type, setup structure and process flexibility, appears modified. For that, through simulation studies, some researchers discussed performance of employing heuristics designed for the classical ELSP in these new situations. We can resume that investigations have focused in testing different policies in situations in which demand type and item type are varied. Concretely production systems with dynamic stochastic demand (Leachman and Gascon (1988), Gascon et al. (1994)) , static stochastic demand (Vergin and Lee (1978), Brander et al. (2005)) and hybrid make-to-order and make-to-stock systems (Soman et al. (2004)) has been simulated. In some cases the same heuristics has been tested with modifications in the input conditions. Vergin and Lee (1978) were the first to propose and test dynamic scheduling policies based on feedback and inventory levels, under varying cost and system parameters. They tested two rules for deterministic demand: classical cyclical production lot size for multiple products (EOQ), EOQ modified incorporating shortages costs and four rules for dynamic scheduling: Magee's Rule and three alterations of this rule whose incorporate respectively maximum inventory level, backorders and elimination many very short production run. Leachman and Gascon (1988) tested four rules for five products with dynamic stochastic demand in a single machine. The rules are the following: a dynamic lengths heuristic proposed for them in this article, a policy based on independent economic manufacturing quantities for each item, a policy based on the Doll and Whybark procedure,

and a policy utilizing the Vergin and Lee (1978) dynamic scheduling rules, involving five items produced on a single machine. At Gascon et al. (1994) six different heuristics for five items with stationary demand with and without forecast errors and dynamic demand are tested. They compare: the Vergin and Lee policy, the lookahead heuristic of Gascon, the dynamic cycle lengths heuristic due to Leachman and Gascon (1988) and the enhanced dynamic cycle lengths heuristic due to Leachman (1991) with two simpler rules: one based on independent economic production quantity and the other based on the Doll and Whybark procedure. Soman et al. (2004) tested four dynamic scheduling policies to modified Bomberger data that include conditions of hybrid MTO and MTS products. Finally, at Brander et al. (2005) appears a simulation study that employs dynamic programming approach from Bomberger and a heuristic method from Segerstedt to calculate lot sizes for four items with stationary stochastic demand. We could generalize that all these authors finished their studies with two main conclusions: that the policies which consider current inventory levels and appropriated decision rule in making scheduling decisions outperform policies whose are based solely on the solutions of an ELSP (deterministic) model, and that the methods that perform well for classic ELSP conditions do not necessarily perform well for ELSP variants.

Through the review of the bibliography we have identified simulation studies which tests scheduling policies for ELSP for different demand and item type. We pretended to propose and simulated different scheduling policies to a new ELSP variant, Part Grouping ELSP (PGELSP). This problem consists in a product system in ELSP environment in which more than one product can be produced at a time on the machine. In order to obtain conclusions a simulation model and their results have been developed in the article, employing modified Bomberger data which include items that could be produce simultaneous. For that, this paper compares with three simpler rules which are modified in order to consider Part Grouping. These heuristics are. EMQ, Doll and Whybark (1973) and Fransoo, (1993). The rationale for including these simply heuristics in the comparison is to better understand the value of added Part grouping in scheduling rules. In the next section we summarize the logic of each heuristic and how each heuristic was implemented for our simulation tests.

## 2. Problem Description

We consider a problem of scheduling items, some of them can be produced at the same time single facility with limited capacity. We make the following assumptions in this paper:

- More than one product,  $i$  or  $i + j$ , can be produced at a time on the machine
- Product production rates are deterministic and constant,  $p_i, p_{ij}$
- Product setup costs and times  $c_i, c_{ij}$  are independent of production order
- Product demand rates are stochastic with mean  $d_i$
- Inventory costs are directly proportional to inventory levels
- Production capacity is sufficient to meet total demand.

- If demand doesn't met in the periods in which occur, a lost sales cost is incurred which are proportional of units lost and cost per unit item.

This problem is named PGELSP, because incorporating Part Grouping in ELSP. The objective is to minimize  $\sum C_i$  by determining the optimal  $T_i$  and  $T_{ij}$  subject to the capacity requirement constraint. The total cost equation for the economic manufacturing quantity incorporating parts grouping is, (1).

$$C_i = \frac{H}{T_i} s_i + \frac{H}{T_{ij}} s_{ij} + h_i \left[ \frac{T_i}{2} q_i \left( 1 - \frac{d_i}{p_i} \right) + \frac{T_{ij}}{2} q_{ij} \left( 1 - \frac{d_i}{p_{ij}} \right) \right] \quad (1)$$

In which is considered that the product i can be produced separately or together the product j,  $q_i$  the part of the demand for product i assigned to the production of I, and  $q_{ij}$  the part of the constant demand for product i assigned to the production of i with j. In case product i cannot produce together with other product j,  $q_i = d_i$ , in other case  $d_i = q_i + q_{ij}$ .

### 3. Scheduling Rules

In this section, we present a brief summary of various scheduling rules modified in order to include part grouping. These methods are basically run-out based scheduling rules, which widely used in industry as they are easy to understand and implement. So, we assume that at the production decision moment, the run-out time,  $RO_i$  for each item is calculated.  $RO_i$  is defined according to Gascon et al. (1994) as the expected duration until the inventory of item i falls to a reorder point equal to safety stocks  $ss_i$  plus the expected demand during the changeover time. So,  $RO_i$  is given as (2):

$$RO_i = \frac{I_i - ss_i}{d_i} - c_i \quad (2)$$

Without loss of generality, items are re-numbered such that:  $RO_1 \leq RO_2 \leq \dots \leq RO_n$ . The first product is then chosen as the product to be produced next.

In order to apply correctly the heuristics, we must to define values of initial and safety stocks of each item. In one hand, we consider that initial inventories are equal to half part of maximum stock for each heuristic. On the other hand, the safety stock levels are determined by deploying the standard textbook method Silver et al., (1998), that uses the demand variance and the desired service levels. So, for service levels of 95%, we can define safety stock, as (3):

$$ss_i = 1.65\sigma\sqrt{T_i(1-d_i/p_i)} \quad (3)$$

in which formula  $\sigma$  standard deviation of demand, and  $T_i$  target cycle according to corresponding heuristic. Making adaptation in to be able to incorporate part grouping in a production cycle, we define (4):

$$ss_i = 1.65\sigma\sqrt{T_i(1-d_i/p_i)} + 1.65\sigma\sqrt{T_{ij}(1-d_i/p_i)} \quad (4)$$

### 3.1. EMQ heuristic modified with Part Grouping

The EMQ heuristic is based on the cycles for independent manufacturing, as showed in (5) and we modified this in order to incorporate part grouping, as showed in (6);

$$T_i = \sqrt{2s_i H / h_i q_i (1 - d_i / p_i)} \quad i = 1, \dots, n \quad (5)$$

$$T_{ij} = \sqrt{2H \left( 1/h_i q_{ij} (1 - d_i / p_{ij}) + 1/h_j q_{ij} (1 - d_j / p_{ji}) \right) (s_{ij} + s_{ji})} \quad i = 1, \dots, n \quad (6)$$

where, H is total annual number of production days of capacity available, and for item  $i=1, \dots, n$ ,  $T_i$  is the target cycle,  $s_i, s_{ij}$  are the cost to setup the process for one lot (batch) of product i, of product i with product,  $p_i$  and  $p_{ij}$  are the daily production rate of product i, or of product i produced with product j,  $h_i$  is the cost to hold one unit in inventory for one year,  $d_i$  is the daily demand for product i,  $q_i$  is the part of the constant daily demand for product i assigned to the production of i, and  $q_{ij}$  is the part of the constant demand for product i assigned to the production of i with j.

In this heuristic, items are produced according to their economic manufacturing quantities, but truncating production runs wherever another item inventory is running out. So, it is basically a multi-item (s,S) policy, in where, being  $ss_i$  is the safety stock, according to (7):

$$s = S \min = ss_i + c_i d_i, \quad S = S \max = ss_i + T_i q_i (1 - d_i / p_i) + T_{ij} q_{ij} (1 - d_i / p_i), \quad (7)$$

So, in this rule the production of the current item continues until inventory of that product reach  $S_{\max}$  or the inventory of another products falls below  $S_{\min}$ .

### 3.2. Doll and Whybark

Our implementation of the dynamic of Doll and Whybark heuristic is relatively similar to the EMQ heuristic, changing way of calculating target cycle, in our case  $T_i$  and  $T_{ij}$ . We implemented a modified version for this rule, that incorporate part grouping. In Doll and Whybark, the target cycle for item i,  $T_i$ , is a multiple of a target fundamental cycle length, T, that is  $T_i = k_i T$ ,  $k_i$  being a positive integer. So incorporating part group, we have also consider  $T_{ij} = k_{ij} T$ . The objective is to find values of T and  $k_i$  that minimize the sum of changeover cost and inventory cost for each item, that is incorporating part grouping, (8):

$$\text{Min} \sum_i C_i = \sum_i \left\{ \frac{H}{k_i T} s_i + \frac{H}{k_{ij} T} s_{ij} + h_i \left[ \frac{k_i T}{2} q_i \left( 1 - \frac{d_i}{p_i} \right) + \frac{k_{ij} T}{2} q_{ij} \left( 1 - \frac{d_i}{p_{ij}} \right) \right] \right\} \quad (8)$$

So, firstly initial estimate of the basic period T are calculated. For that,  $T_i$  and  $T_{ij}$  are calculated according to (5), and (6) for each item, and T is selected as a smallest value of these, i.e.,  $T = \min \{ T_i, T_{ij} \}$ . Then, the  $k_i$  and  $k_{ij}$  values are selected as the closest power-of-two integer multiple (rounded up or down) to  $T_i / T$ , and  $T_{ij} / T$  that incurring less value for

function  $C_i$ . At this point, the basic period time  $T$  are recomputed using the new estimates of  $k_i$ , according to (9):

$$T = \sqrt{2H \left( \sum_i \alpha_i s_i + \sum_{ij} s_{ij} \right) / \left( \sum_i h_i q_i (1 - d_i / p_i) + \sum_{i,j} h_i q_{ij} (1 - d_i / p_{ij}) \right)} \quad (9)$$

where  $\alpha_i$  is a factor which is equal to 1 if it is adequate that item  $i$  is produced separated, and 0 if it is appropriated part grouping. With this value of  $T$ ,  $k_i$  estimations are recalculated. The procedure terminates when consecutive iterations produce identical values of  $k_i$ . Then, values of  $T_i$  are calculate for each item  $i$  as  $T_i = k_i T$  and  $T_{ij} = k_i T$ .

### 3.3. Fransoo

Fransoo, (1993) suggests a simple policy aimed at achieving. The idea is to stick to target cycle times as much as possible. In this case, the production quantity of the product chosen for the production is not affected by the event of some other product running out. In high utilization case, this may save the number of setups and hence the productive capacity but at the same time some orders may be lost. Based on the run-out times, a product  $i$  with  $\min RO_i$  is indexed as 1 and is selected for the production and the production quantity is since their reaches  $S_{\max}$ , given as (10):

$$S = S_{\max} = ss_i + T_i q_i (1 - d_i / p_i) + T_{ij} q_{ij} (1 - d_i / p_i) \quad (10)$$

with  $T_i$  and  $T_{ij}$ , calculated according our modification of Doll and Whybark (1973).

## 4. Simulation Model

A simulation model is developed using Anylogic 6.0 to evaluate the performance of Part Grouping under different scheduling heuristic. The model has two main modules—an order generator module that generates the orders based on the demand distribution, and a shop floor control module that contains the shop configuration under investigation and the various scheduling rules to operate the shop.

### 4.1. Model dynamics

The “target cycle” times are pre-calculated either using (a) EMQ modified incorporating Part grouping, or (b) Doll and Whybark also modified incorporating Part grouping. Safety stock  $ss$  and order up-to levels  $S_{\max}$  for MTS products are pre-calculated based on the mean and standard deviation of the demand during replenishment lead-time and desired service level. Initial stocks are considered half part of  $S_{\max}$ . These target cycle times, safety and initial stocks, and order up-to levels are used as inputs at the operational decision level.

The timing sequence in the simulation model is as follows.

1. At the beginning of the period, the demand for each item is generated. The demand is fulfilled from the stock. The inventory balance is updated. If demand cannot be met, it is lost.
2. At the end of each production run, the run-out times are calculated for all the products and the one with the smallest run-out time is selected for the next production run.

- The production start times and the production quantities are calculated based on the scheduling heuristic rule chosen.

We consider that a period is a day. For each scheduling heuristic, a simulation run lasts for 240 periods are done. In case that more than one product, which can be produced separately or with other at the same time, we consider option that incurred in less cost in theory.

#### 4.2. Conditions of experimentation

All the simulations were run on a year horizon assuming ten items are produced on a single machine. Production activity was assumed 240 days in a year, only on weekdays. To evaluate and compare the scheduling rules, discussed in the earlier section, we use the Bomberger data set, which is most commonly used in ELSP literature (e.g. Haessler (1979)). In order to incorporate parts grouping these data are modified as shown in Table 1.

**Table 1.** Bomberger Data Set modified

Part N. Bomberger	Setup Cost	Unit Cost*	Prod Rate (unit /day)	Demand** (unit/day)	q (unit/day)	Setup Time (hours)
1	15	0,0650	30000	400	400	1
2	20	0,1775	8000	400	0	1
3	30	0,1275	9500	800	325	2
2(with 3)	25	0,1775	4000		400	1,5
3(with 2)	25	0,1275	4750		475	1,5
4	10	0,1000	7500	1600	1600	1
5	110	2,7850	2000	80	80	4
6	50	0,2675	6000	80	80	2
7	310	1,5000	2400	24	24	8
8	130	5,9000	1300	340	119	4
9	200	0,9000	2000	340	0	6
8(with 9)	165	5,9000	650		221	2
9(with 8)	165	0,9000	1000		340	3
10	5	0,0400	15000	400	400	1

\*Annual inventory cost = 10% of item cost and one year = 240 - 8 hour days

\*\*Normal distribution, coefficient of variance 0,1

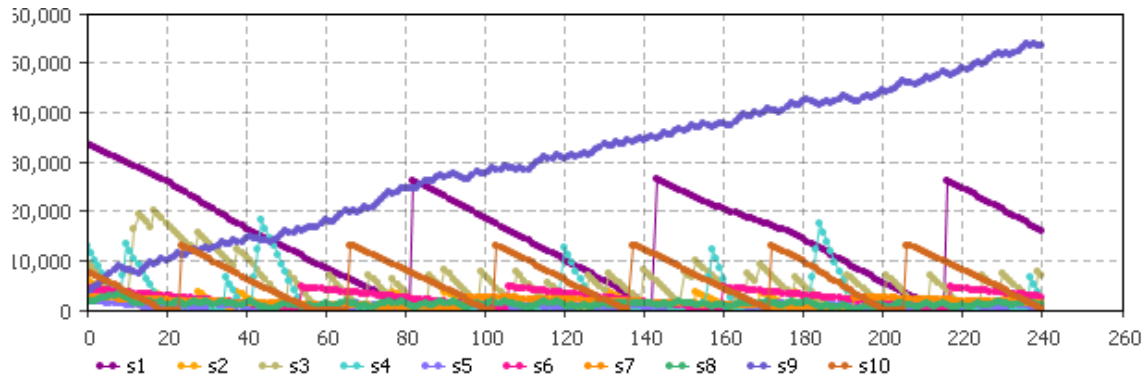
Lsst Sales Cost= 10% of item cost

We decided to incorporate parts grouping in part number 2 and 3 because are the first with all the values different. Product 8 and 9 are chosen because of their symmetry with 2 and 3. The values of parameters for the parts grouping are needed and created according to these rules. We consider setup cost, setup time and product rate  $(s_{ij}, ts_{ij}, p_{ij})$  are reduced when items are produced simultaneous. Concretely, we supposed are half part of the value when are just produced,  $s_{ij} = (s_i + s_j) / 2$ ,  $ts_{ij} = (ts_i + ts_j) / 2$ ,  $p_{ij} = p_i / 2$ . However, item cost is assumed to stay the same as parts groping is done and not.

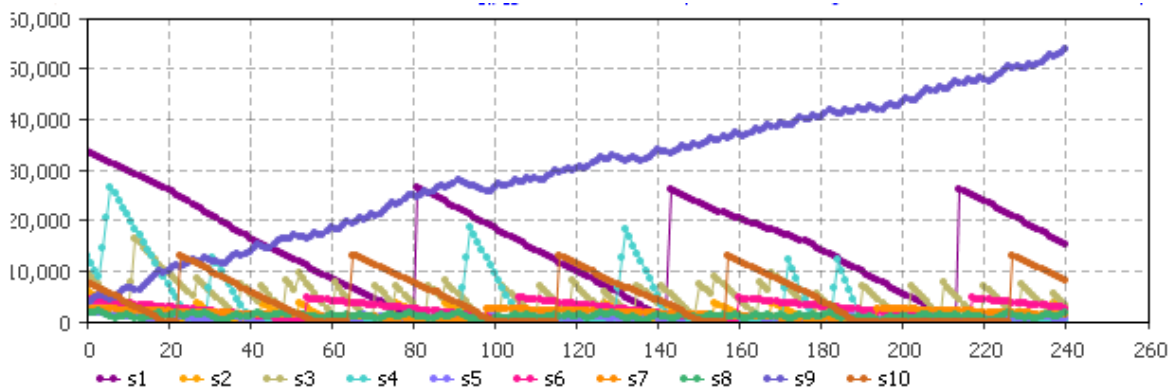
We decided to include the lost sales cost as it gives an indication of service levels for fulfilled demand. We chose a modest value of 10% of item cost, the same value of holding costs. The demand rate shown in this table is for the case where utilization is 88%. As table 1 shows, according to values of  $q$  we established that product 2 always will be produced with product 3, and product 8 always will be produced with product 9. We supposed this assumption because in theory supposed less cost that produce each item separately. With simulation results we can tests behaviour of this assumption.

#### 5. Simulation Results and Analysis

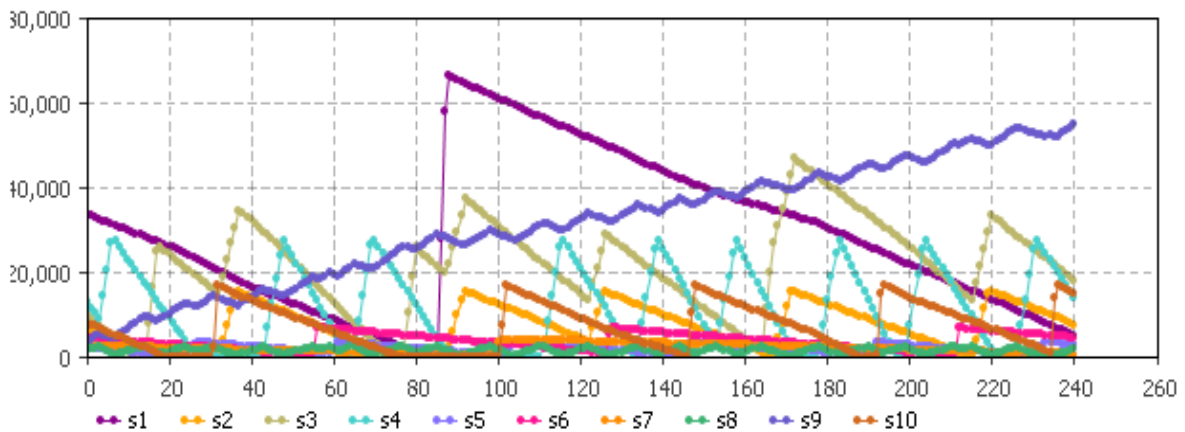
Figure 3, shows the inventory of the ten items subject to the different scheduling rules. In Table 2 costs of setup, holding and lost sales are showed for each heuristics.



(a) EMQ



(b) Doll and Whybark



(c) Fransoo

**Figure 3.** Impact of Part grouping addition on stocks

We can observe in Figure 3 how inventory of item 9 becomes higher for all the heuristics due to is always produces with item 8 which a low production rate. Stocks for rest of items present similar evolution in heuristic EMQ and Doll and Whybark. It is in Fransoo heuristic in which we can observe different evolution due to reduction of change of orders. If we analyse Table 1, we obtained the conclusion that is Fransoo heuristic the scheduling rule which performs better for this case. Another time results for heuristics of EMQ and Doll and Whybark are similar.

**Table 2.** Impact of Part grouping addition on costs

<b>Costs</b>	<b>Lost Sales</b>	<b>Holding</b>	<b>Setup</b>	<b>Total</b>
EMQ	9.581,947	6.352,182	24.765	<b>40.699,129</b>
Doll and whybark	10.962,007	6.424,412	24.130	<b>41.516,419</b>
Fransoo	9.583,815	7.689,042	10.480	<b>27.752,857</b>

## 6. Conclusions and future research

We pretended to propose and simulated different scheduling policies to a new ELSP variant, Part Grouping ELSP (PGELSP). This problem consists in a product system in ELSP environment in which more than one product can be produced at a time on the machine. In order to obtain conclusions a simulation model and their results have been developed in the article, employing modified Bomberger data which include items that could be produce simultaneous. For that, this paper compares with three simpler rules which are modified in order to consider Part Grouping. These heuristics are. EMQ, Doll and Whybark (1973) and Fransoo, (1993). For the experiments done seems that Fransoo heuristic is the most appropriated scheduling rules, it could be due to we are working in a facility with high level of utilization. In order to obtain better results we can improve way of choosing if the product is produced separately or joined with other product, in case this option is possible.

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