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Part grouping in ELSP

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1. Introduction

Scheduling the production of several products with deterministic and constant demand on a single facility, with the objective of reducing the sum of holding costs and setup costs, has been studied in the literature under the name of ELSP- Economic Lot Scheduling Problem.

The economic lot scheduling problem has been investigated extensively in the literature over the years. A comprehensive review until 1976 can be founded in Elmaghraby (1978), who reviews various approaches to the problem and divides these into two categories; analytical approaches that achieve the optimum for a restricted version of the original problem; and heuristic approaches that achieve ‘‘good’’ solutions for the original problem and more recent work in Silver et al., (1998).

In many industries, especially in the stamping industries, the flexibility of the process Sheti and Sheti (1990) allow that more than one product can be produced at a time on the machine. In this case, appears the option that a machine can produce an item i, an item j or an item i, and j simultaneous. This adds another dimension to the ELSP because the problem is obtained in which is the production lot size of each combination which minimizes the total costs of setup and holding. To analyze in which conditions is profitable grouping parts or not, this study is done.

2. Problem Description

The classic problem ELSP will be characterized by the following: one product can be produced at a time on the machine, product production rates are deterministic and constant, product setup costs and times are independent of production order, product demand rates are deterministic and constant, demand must be met in the periods in which occur, and inventory costs are directly proportional to inventory levels production capacity is sufficient to meet total demand.

- This paper consider option that more than one product can be produced at a time on the machine, so part grouping (PG) is allowed in classical ELSP (PGELSP). In order to analyze in which conditions PG is appropriate, following notation is employed:

d_i Constant daily demand for product i

p_i, p_{ij} Daily production rate of product i, or of product i produced with product j

s_i, s_{ij} Cost to setup the process for one lot (batch) of product i, of product i with product

h_i Cost to hold one unit in inventory for one year

C_i Total cost per year for product i

TC Total cost per year for all the products

T_i, T_{ij} Cycle time or number of time units between consecutive production runs of product i or product i produced with product j

q_i , q_{ij} Part of the constant demand for product i assigned to the production of i, and to the production of i with j.

TP_i, TP_{ij} Total annual processing and setup days needed for product i, or for product i produced with product j

TT Total annual number of production days of capacity available

Historically, the economic order quantity (EOQ) is presented by Harris (1913) predates ELSP problem. EOQ is also known as the Wilson lot size formula since it was used in practice by Wilson (1934) EOQ balances the setup cost and inventory holding cost. In the EOQ model, demand is known with stationary rate and the planning horizon is infinite. The total cost equation for the economic manufacturing quantity incorporating part grouping is, (1):

$$C_i = \frac{H}{T_i} s_i + \frac{H}{T_{ij}} s_{ij} + h_i \left[\frac{T_i}{2} q_i \left(1 - \frac{d_i}{p_i} \right) + \frac{T_{ij}}{2} q_{ij} \left(1 - \frac{d_i}{p_{ij}} \right) \right] \quad \textbf{(1)}$$

In which is considered that the product i can be produced separately or togheter the product j. The objective is to minimize $\sum C_i$ by determining the optimal T_i and T_{ij} ,subject to the capacity requirement constraint.

In this paper we proposed analyzed in which conditions is interesting produced in groups or not. So we analyze the effect of changing various parameters such as holding cost, setup cost and demand and production rate. Several authors have analyzed versions of the ELSP in which these characteristics have been modified. For example, we can find in the literature the case of lot scheduling with stochastic demand in Sox et al. (1999), variable production rate in Eynan (2003), restrictions on the capacity Campbell and Mabert (1991), setup costs dependent of production order in Dobson (1992) or backorders. However, the ELSP variant that include the possibility that more than one product can be produced at a time on a machine, have not considered. So, our investigation has focused in this option in the ELSP environment.

In our study we presented a simple case of two products A and B, which can be produced together or not in a machine. We modify their respective values of setup and holding costs, demands and production rates in order to evaluate in which conditions part grouping is appropriate. Through different figures, which shown the lot size percentage of each combination and the cost performance, the conclusions of the analysis can be obtained.

3. Analysis Model

A model is developed (using Mathematica 6.0) to evaluate the adequacy of part grouping under different situations. The model has three main modules: an input of data in which corresponding parameters are modified, a central module which calculate the solution with low cost, and a third module which shows results. In the central module cost of three production options are evaluated, which are:

1. To produce each product separately, as if the possibility of part grouping does not exist. In this case we are in the case of applying EMQ formula. This option is named EMQ.

2. To produce together both products, and separately one of them, according relations between product rate and demand of each product. This option is named Pure Mix.

3. To produce together and separately both product. This option generates a large number of suboptions. This option is named Mix.

There are various data sub-modules to measure the performance of the production options. We are interested in the cost saving when parts groping are the best choice, the part of the demand met in each process, and the time cycle associated to this. Each of these models is developed with a lot of flexibility and can be customized to mimic real life situations to the greatest extend possible.

3.1. Model dynamics

For the experimental analysis we consider the case of the production of two products A and B, which can be produced or separately or together. The value of the data and the initial value of modified parameter are inserted. Then, program start searching variables $\left(T_a, T_b, T_{ab}=T_{ba}, q_a, q_b, q_{ab}, q_{ba}\right)$ values that optimize this nonlinear problem, (2):

$$Minimize\ CT = Minimize\left\{\begin{array}{l} \frac{T_a}{2} q_a h_a\left(1-\frac{d_a}{p_a}\right)+\frac{H}{T_a} s_a+\frac{T_b}{2} q_b h_b\left(1-\frac{d_b}{p_b}\right)+\frac{H}{T_b} s_b+ \\ +\frac{T_{ab}}{2}\left[q_{ab} h_a\left(1-\frac{d_a}{p_{ab}}\right)+q_{ba} h_b\left(1-\frac{d_b}{p_{ba}}\right)\right]+\frac{H}{T_{ab}}\left(s_{ab}+s_{ba}\right) \end{array}\right\}$$

subject to;

$$d_a=q_a+q_{ab}, \quad d_b=q_b+q_{ba}, \quad q_{ab}=q_{ba}\left(\frac{p_{ab}}{p_{ba}}\right) \tag{2}$$

3.2. Experimental factors

To evaluate and compare in which situations is adequate produce together or not, we use an extract to the Bomberger dataset (Bomberger (1966)) which is most commonly used in ELSP literature. We select Bamberger's part number 2 and 3, because they are the firsts with the values of setup cost, piece cost, product rate, demand and setup time different between them, see Table 6.

Table 6. Bomberger Problem

	Part N. Bomberger	Setup Cost	Unit Cost*	Prod Rate (unit /day)	Demand (unit/day)	Setup Time (hours)
	1	15	0,0650	30000	400	1
A	2	20	0,1775	8000	400	1
B	3	30	0,1275	9500	800	2
	4	10	0,1000	7500	1600	1
	5	110	2,7850	2000	80	4
	6	50	0,2675	6000	80	2
	7	310	1,5000	2400	24	8
	8	130	5,9000	1300	340	4
	9	200	0,9000	2000	340	6
	10	5	0,0400	15000	400	1

*Annual inventory cost = 10% of item cost and one year = 240 - 8 hour days

We can observe that these items have not very different values of setup and holding cost. Respect to demand and product rate, comparing their quotient, we can affirm the same. Therefore, when results of sensibility analysis are discussed, we could be interesting change them for very different values to confirm conclusions. Also, the demand of product A is lower than then demand of product B. So when we change parameters we have to refer is this alteration concern the product with a low or high demand. Values of parameters for the part grouping are needed and created according to these rules. We consider setup cost, setup time and product rate $\left(s_{ij}, p_{ij}\right)$ are reduced when items are produced simultaneous. Concretely, we supposed are half part of the value when are just produced, $s_{ij} = \left(s_i + s_j\right)/2, p_{ij} = p_i/2$.

4. Experimental Results and Analysis

For each experiment, three figures and one table are showed and analyzed. The first figure shows, at different values for the parameter modified the relation between: the part of the constant demand (unit/day) for product i assigned to the production of i separately and the total demand for product i, q_i/d_i, and the part of the constant demand (unit/day) for product i assigned to the production of i with j and the total demand for product i, q_{ij}/d_i, for the products named A and B. So, four curves appear in each figure.

The second figure shows, at different values for the parameter modified: the cycle time for product i when it is produced separately, T_i, and the cycle time for product i when it is produced with product j, $T_{ij} = T_{ji}$,for the products named A and B. So, three curves appear in each figure. The third combined figure shows, at different values for the parameter modified, the cost total in the case of part grouping is not allowed, the cost total obtained solving the problem in the environment of study (part grouping is allowed), and the difference between these values for the product A and B. There are two curves in the top sub-figure. The difference in “EMQ Cost” and “Total Cost” curves in each figures represent the decrease of cost obtained because of the addition of the possibility of part grouping, which is showed in the below sub-figure . Finally, another graph are shown in order to extend this analysis from differents relations between all the variables (holding cost, setup cost and demand&product

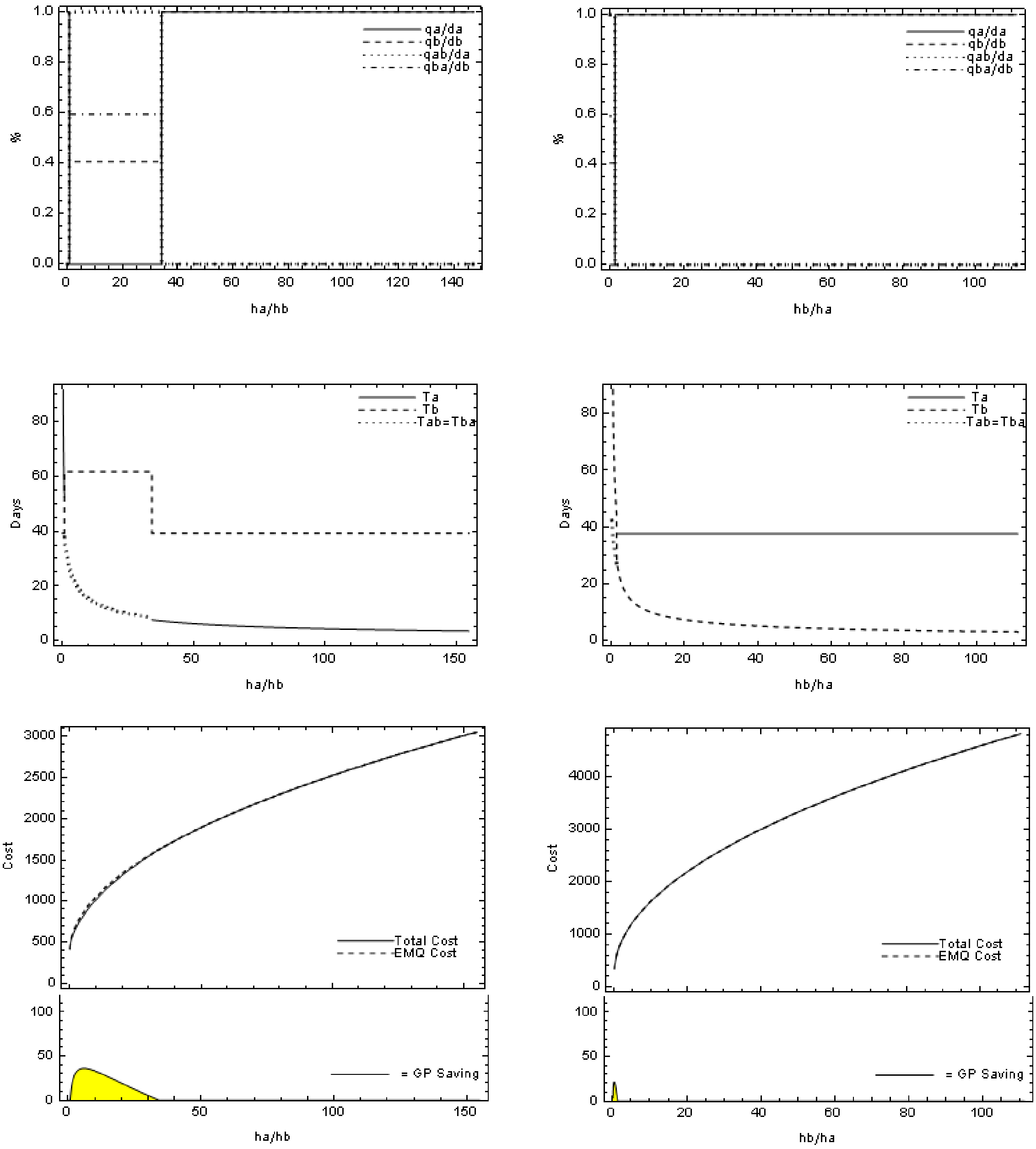
rate), It is a 3d graphic which show the best productive option, according to values of h_a/h_b, s_a/s_b and d_a/d_b.

4.1. Holding Cost

At this simulation, on one hand, maintaining constant holding cost of product B (0.01275 \$/day), holding cost of product A, which has a lower demand (400 units/day), is modified from $h_a = 0.00075$ to $h_a = 1.975$ with increase of 0.001 units. On other hand, maintaining constant holding cost of product A (0.01775 \$/day), holding cost of product B, which have a higher demand (800 units/day) is modified in an equal manner that A in preview analysis. At the first analysis situation, when values of h_a are modified, we can appreciate on the left of the

Figure **4**, that part grouping only compensate for h_a between 0.009 and 0.435, which correspond to $h_a/h_b \in [0.733, 34.110]$. In the third graph we can observe saving of employing part grouping which in this case is not very significant, about 6% in the best situation. At the second analysis, when values of h_b are modified, we can appreciate on the right of the

Figure 4, that part grouping only compensate for h_b between 0.0007 and 0.056, which correspond to $h_b/h_a \in [0.042, 1.338]$. In the third graph we can observe saving of employing part grouping which in this case is not very significant, about 7% in the best situation. In both situations when part grouping compensate, which is a minority, is adequate produce B, and A+B, that is pure mix.



(a) Modifying h_a

(b) Modifying h_b

Figure 4. Impact of holding cost

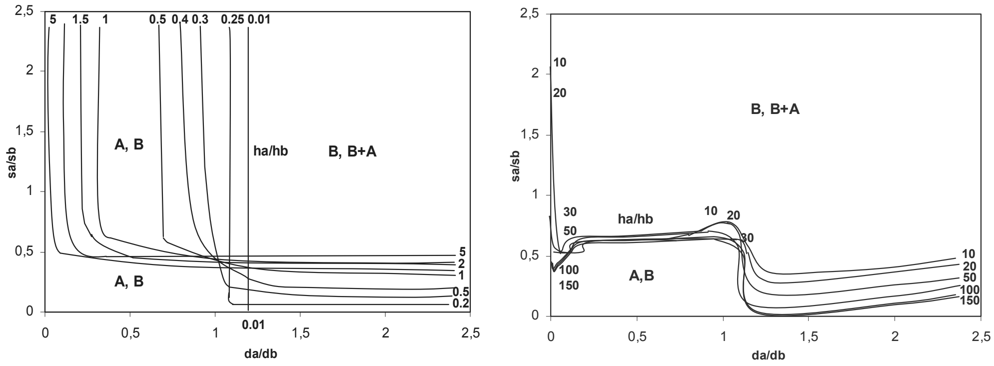


Figure 5. Evolution of Part Grouping according to holding cost, demand and setup cost

At the first analysis situation, when values of h_a are modified, we can appreciate on the left of the

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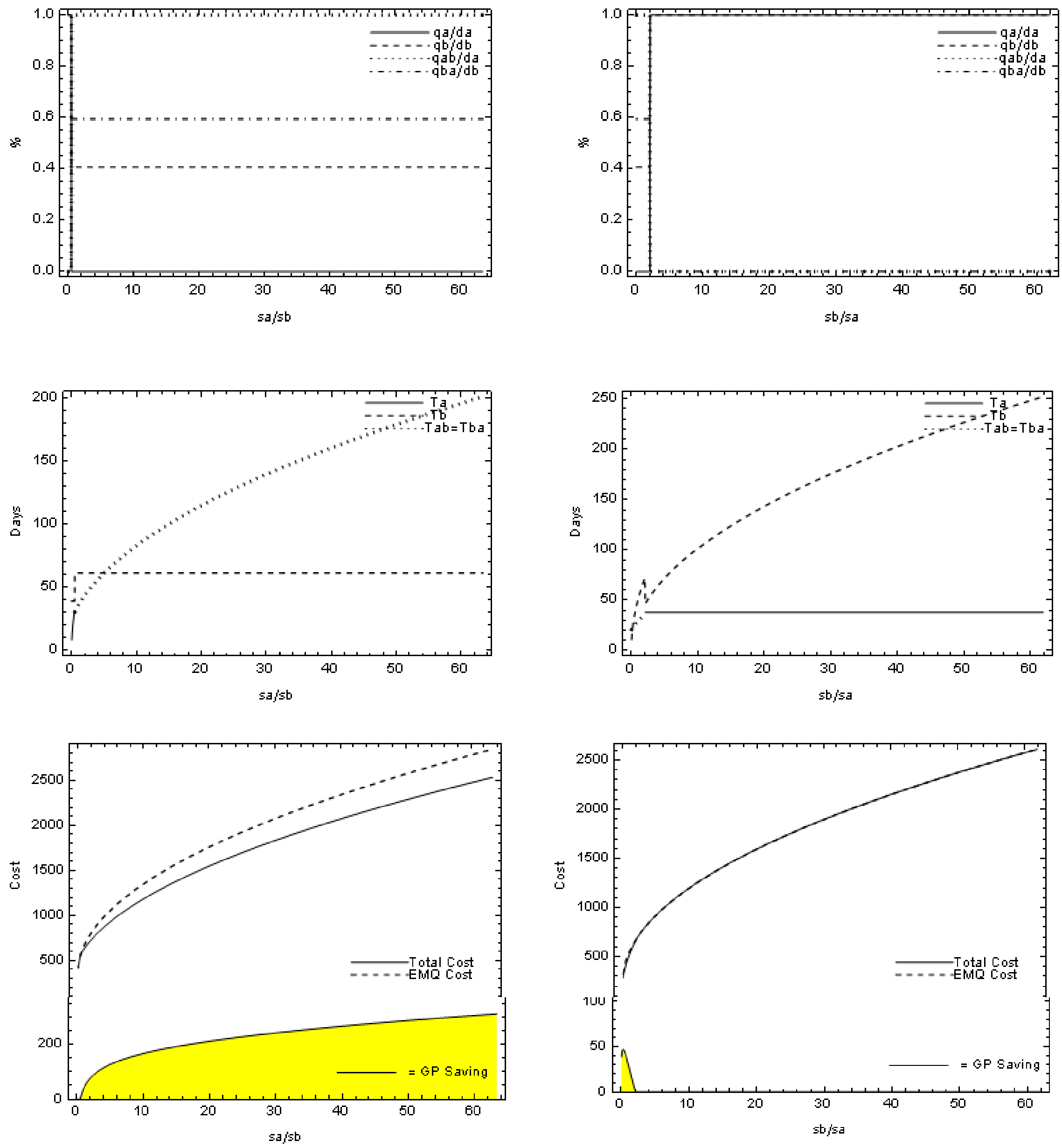
In order to achieve a complete vision of effects of holding cost and part grouping, we propose Figure 5, which presents for different low relations between demand and setups, when part grouping is the best option.

4.2. Setup Cost

At this simulation, firstly, maintaining constant setup cost of product B (30$), setup cost of product A, which has a lower demand, is modified from $s_a = 1$ to $s_a = 1900$ with unitary increase. Secondly, maintaining constant setup cost of product A (20$), holding cost of product B, which have a higher demand is modified in an equal manner that A. At the first analysis situation, when values of s_a are modified, we can appreciate on the left of the Figure 6, that part grouping compensate in the majority of cases. Only for s_a between 0. 999 and 14, which correspond to $s_a/s_b \in [0.0333, 0.4670]$, not Part grouping is the best choice. For values of s_a higher than 14, produce B, and A+B is the most adequate, adding saving since 12%.



(a) Modifying s_a

(b) Modifying s_b

Figure 6. Impact of setup cost

At the second analysis, when values of s_b are modified, we can appreciate on the right of the Figure 6, that part grouping only compensate for s_b between 1 and 41, which correspond to $s_b/s_a \in [2.1, 62]$. In the third graph we can observe saving of employing part grouping which in this case is about 10% in the best situation. So, we can observe that the best option productive is very different depending of the value of s_a *or* s_b, which corresponding with items with minor and major demand respectively.

In order to achieve a global vision of effects of setup cost and part grouping, we propose Figure 7, which presents from setup cost perspective when part grouping is the best option for different relations between demand and holding cost. We can observe that parts groping is ever appropriate when relations of demand and are higher than 1. For relations of setups between 0.4 and 0.75, not part grouping is and option with an important status, mainly when this relation is growing up to 0.75. For the rest of values of setup cost, which includes relation lower than 0.4 and higher than 0.75, except for relation of holding cost lower than 5, pure mix is without doubts best productive option.

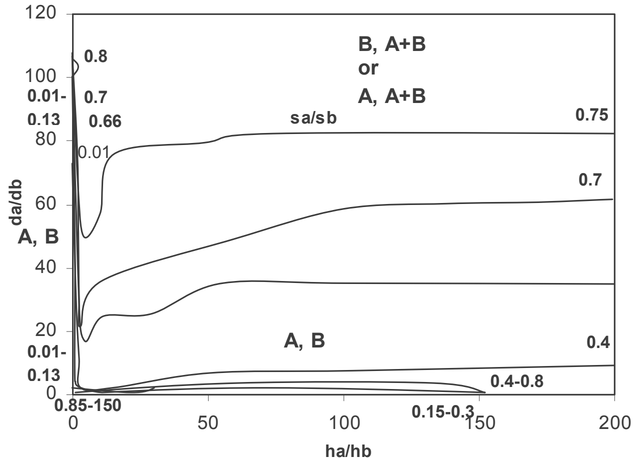
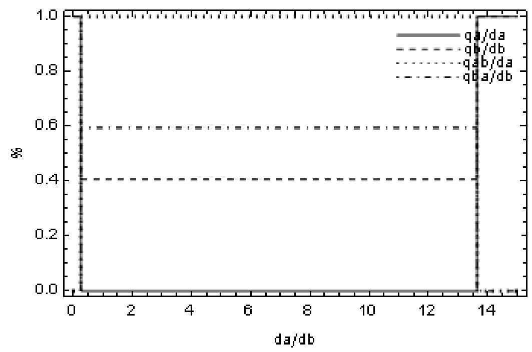
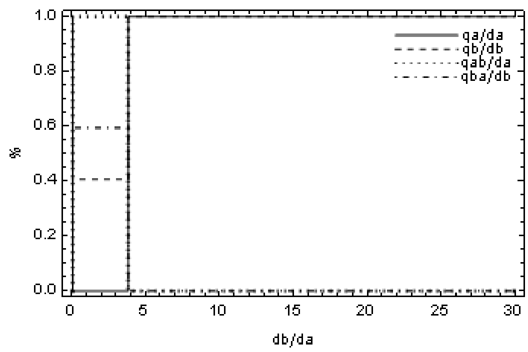


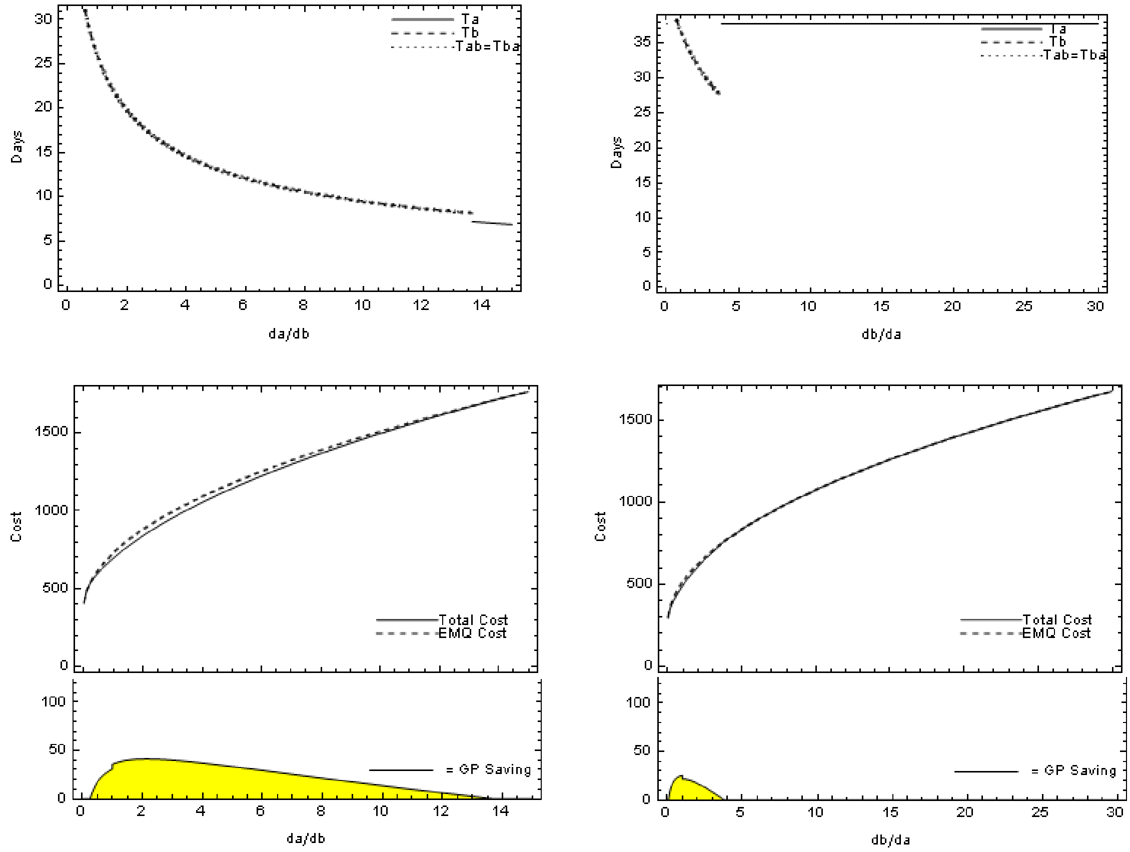
Figure 7. Evolution of Part Grouping according to setup cost, demand and holding cost

4.3. Demand and Product Rate

At this simulation, on one hand, maintaining constant demand and product rate of product B (800 and 9500 units/day respectively), demand and product rate of product A is modified. Demand is varied from $d_a = 12$ to $d_a = 12000$ with unitary increase, while production rate p_a is modified so that relation between demand and product rate, d_a/p_a, is constant in all the simulation and equal to the original situation, $d_a/p_a = 400/8000 = 0.5$. On other hand, maintaining constant demand cost of product A, demand and product of product B, is modified in an equal manner that A, except that in this case relation between demand and product rate is 0.08421.







(a) Modifying d_a

(b) Modifying d_b

Figure 8. Impact of demand

At the first analysis situation, when values of d_a are modified, we can appreciate on the left of the Figure 8, that part grouping compensate in the majority of cases. Only for d_a between 12 and 209, and between 10960 and 12000 which correspond to $d_a/d_b \in [0.015, 0.261 \cup 13.7, 15]$, not Part grouping is the best choice. For the rest of values of s_b , produce B, and A+B is the most adequate option, adding saving since 5%. At the second analysis, when values of d_b are modified, we can appreciate on the right of the Figure 8, that part grouping only compensate for s_b between 30 and 1528, which correspond to $d_b/d_a \in [0.075, 3.82]$. So, we can observe that the best option productive is very different depending of the value of d_a *or* d_b, which corresponding with items with different values of holding and setup costs.

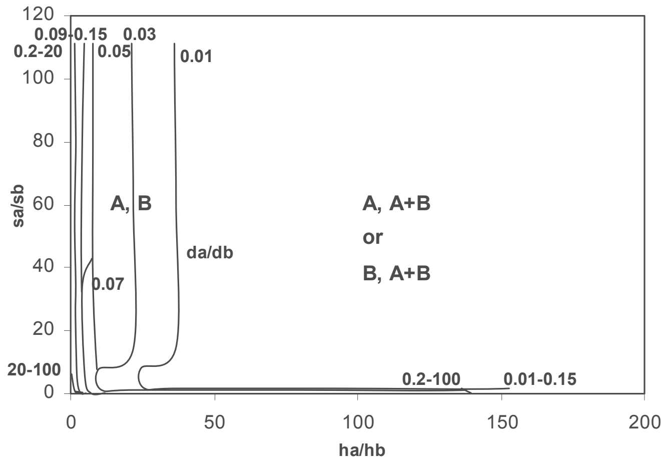


Figure 9. Evolution of Part Grouping according to demand, holding and setup cost

In order to achieve a global vision of effects of modifying demand, we propose Figure 9, which presents from demand perspective when part grouping is the best option for different relations between setup and holding cost. For relations of holding cost h_a/h_b higher than 40 and s_a/s_b higher than 5, produce B, and A+B is the best productive option, independently of values of demand. Only for values of demand between 0.01 and 0.09 not part grouping is adequate, as long as h_a/h_b is lower than 40. .

5. Conclusions and future research

The possibility of more than one product can be produced at a time on a machine in the environment of the ELSP, in despite of being common, in practice is not addressed in the literature. In this article we try to start filling this gap. We analyse for different situations in which conditions results profitable part grouping for two products A and B.

Firstly we make analysis with items A and B which corresponds to item two and three of Bomberger data, for being first of these with all parameters different. In this way, three series of experiments are made, modifying in first place holding cost maintaining setup cost and demand constant and invariable, in second place setup cost maintaining holding cost and demand constant and invariable and finally demand maintaining holding and setup cost constant and invariable. Results indicate that sometimes Part grouping is the best productive option, achieving saving as 12%. Depending of the values of parameters could be interesting not part grouping, producing product A and product B separately. In part 4 of their paper this results are detailed, but it's very important to for the wide interval of values of holding cost, setup cost, and holding cost for the three series of experiment never producing A, B and A+B is profitable. This productive option is named Mix, and never conduced to a minimal cost compared to the option of produce A, and B (EMQ) or A and A+B, or B and B+A a kind of part grouping named Pure Mix.

In order to extract global conclusion, we extended parameter of A and B, to a wide range of values of demand, setup and holding cost, showing graphs three-variable. Also, graph could be observed of three view, holding cost in Figure 5, setup cost in Figure 7 and demand in Figure 9. We obtained for the wide interval of values of holding cost, $h_a/h_b \in [0.01,150]$, setup cost, $s_a/s_b \in [0.01,100]$ and holding cost, $d_a/d_b \in [0.01,100]$, that never producing A, B and A+B is profitable. This productive option is named Mix, and never conduced to a minimal cost compared to the option of produce A, and B (EMQ) or A and A+B, or B and B+A a kind of part grouping named Pure Mix.

So, we could conclude that part grouping is an adequate option productive which should be considered, and integrate in the production process. Areas of further research could also consider that setup of part grouping are not reduced, as $s_{ab} = (s_a + s_b)/2$, , but rather increasing, maintained equal or reducing in other manner. It could be interesting study which occurs when the relation between demand and product rate is not constant. Part grouping or more than two products could also be considered.

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