

A survey of parallel hybrid applications to the permutation flow shop scheduling problem and similar problems

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1. Introduction

Parallel algorithms have focused an increased interest due to advantages in computation time and quality of solutions when applied to industrial engineering problems. This communication is a survey and classification of works in the field of hybrid algorithms implemented in parallel and applied to combinatorial optimization problems similar to the permutation flowshop problem with the objective of minimizing the makespan, F_{\max} according to the Graham notation, the travelling salesman problem (TSP), the quadratic assignment problem (QAP) and, in general, those whose solution can be expressed as a permutation*.

2. Parallel hybrid algorithms

It has been demonstrated that collaboration between metaheuristics leads to better quality solutions and more robust methods, Talbi (2002), Basseur et al. (2004). This kind of collaboration between metaheuristics is known as hybrid algorithms, see Bachelet et al. (1996).

We will classify parallel hybrid algorithms according to the type of single algorithms combined: Algorithms that combine only metaheuristics and algorithms that combine exact and approximate methods.

2.1. Hybrid algorithms combining approximate methods

In the field of hybrid algorithms, most work is devoted to this kind of parallel methods. There are several surveys in this field, including those of Talbi (2002), Cotta et al. (2005) and Raidl (2006). These works show a great collection of hybrid methods but most of them are sequential algorithms. We will expand here (table 1) the survey carried out by Loiola et al (2007), paying attention to parallel hybrid methods. (SA: Simulated annealing, GA: Genetic Algorithm, LS: Local Search, ACO: Ant Colony Optimization, TS: Tabu Search).

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Methods	Reference	Problem
SA+GA+LS	Nearchou (2004)	Fm prmu Cmax
GA+SA	Huntley and Brown (1991)	QAP
	Talbi, Hafidi et al. (1998)	QAP
GA+LS	Huntley and Brown (1996)	QAP
ACO+TS	Talbi et al (2001)	QAP
GA+TS	Matsumura et al. (2000)	TSP
Several	Zobolas et al. (2008)	Fm prmu Cmax

Table 1. Parallel hybrid methods combining only approximate methods

We would like to focus our attention in two works devoted to the Fm|prmu|Cmax problem.

The first one is a complex one by Zobolas et al. (2008). It combines four constructive heuristics (NEH, Nawaz, et al. (1983), CDS, Campbell et al. (1970), Palmer (1965) and Gupta (1971)) with two metaheuristics (VNS, Hansen & Mladenovic (2001) and a genetic algorithm, Holland (1992)). It reaches high quality solutions even using a sequential design.

Another sequential hybrid algorithm is the one presented in Nearchou (2004). The author adds features of genetic algorithms and local search to a simulated annealing algorithm. The results regarding the quality of solutions are of the same order of the best ones in literature applied to Taillard (1993) benchmark. We refer in (table 2) the quality of these solutions.

	Jobs	Problem											
		20	20	20	50	50	50	100	100	100	200	200	500
Reference	Machines	5	10	20	5	10	20	5	10	20	10	20	20
Yamada & Reeves (1997)	Hybrid GA						-0.01			-0.24		0.04	
Stutzle (1998)	MMAS	0.41	0.59	0.41	0.15	2.19	2.48	0.20	0.93				
Esquivel et al. (2002)	MCMP-NEH	0.18	0.64	0.54									
Esquivel et al. (2002)	AC-NEH	0.21	1.17	0.86									
Ying & Liao (2004)	ACS	1.19	1.70	1.60	0.43	1.89	2.71	0.22	1.22	2.22	0.64	1.30	1.68
Nearchou (2004)	Hybrid SAA	0.14	0.18	0.14	0.06	0.33	1.78	0.17	0.33	2.11	0.41	2.04	1.41

Table 2. ARPD of solutions obtained by hybrid sequential methods combining only approximate methods

2.1.1. Parallel algorithms

Regarding parallel algorithms combining only approximate methods applied to permutation flowshop or similar problems, we resume them in (table 3), showing the main methods applied. (GA/EA: genetic algorithms/evolutionary algorithms).

Matsumura et al. (2000) describe two types of parallel metaheuristics:

The first one is called cPTS (Cooperative Tabu Search). In this case, solutions obtained via tabu search are recombined using concepts taken from simulated annealing using a so called EX (edge recombination crossover) operator. The innovation of this method is to select the processes that combine their solutions to obtain a new one using different methods. The first, takes from all processes, the one with the best solution. The second selects two random processes and pick the best solution from these two processes.

Method\Method	GA/EA	SA
TS	Matsumura et al. (2000)	Mack et al. (2004)
LS	Denzinger & Offerman (1999)	
SA		Wodecki & Bozejko (2002)

Table 3. Parallel hybrid methods combining only approximate methods

The second one is called HPMH (Hybrid Parallel MetaHeuristic). This method employs only one process for each metaheuristic, one for tabu search and one for the genetic algorithm. The genetic algorithm starts the conversation, sending the best solution to tabu search process when a chromosome is dominating the whole population and prevents genetic algorithm to improve. Tabu search process replies sending the best so far solution as a starting point for a new population in the genetic algorithm process.

Mack et al. (2004) combine simulated annealing with tabu search in parallel to solve the container loading problem. The solution to this problem can be presented as a packing sequence so is similar to the permutation flowshop. The authors begin with a sequential simulated annealing and observed an improvement in quality with respect to tabu search method. Later, they used a hybrid algorithm combining tabu search and simulated annealing, obtaining better quality solutions. The last step was to implement this hybrid algorithm in parallel, to improve even more the quality of solutions.

2.2. Hybrid algorithms combining exact and approximate methods

There are very few references describing applications of hybrid algorithms combining exact and approximate methods. Those applied to permutation problems are only a few of them. We will begin citing some of the sequential algorithms in literature. The only exact method employed is branch and bound.

In (table 4) we have collected some of the works that describe sequential hybrid algorithms combining metaheuristics and exact methods. We have included not only parallel applications but also distributed ones when applied to the $Fm|prmu|Cmax$ and similar problems.

Methods	B&B	Problem
TS	Glover et al. (1996) Woodcock & Wilson (2005)	$Fm prmu Cmax$
GA	Nagar et al. (1996) French et al. (2001)	$Fm prmu Cmax$
LS	Haouari & Ladhari (2003) Framinan & Pastor (2008)	$Fm prmu Cmax$

Table 4. Sequential hybrid methods combining approximate and exact methods

Haouari & Ladhari (2003) insert a branch and bound algorithm inside a local search one to obtain solutions to the permutation flowshop problem. In the next table (table 5) we have compiled some results of hybrid methods applied to instances of well known Taillard's (1993) benchmark.

(nxm)	20 x 5	20 x 10	20 x 20	50 x 5	50 x 10	50 x 20	100 x 5
Instance	ta001	ta011	ta021	ta031	ta041	ta051	ta061
TAI	1278	1582	2297				
NS				2724	3025	3875	5493
LH	1278	1582		2724	2291		5493
t	<1	45		<1	111		<1
BLS	1278	1582		2724	2991		5493
t	<1	67		<1	2994		<1
HPSO	1278	1582	2297	2724	3034	3923	5493
t	1,9	3,7	7,5	29,8	249,9	560,6	353,4
BDS	1278	1582	2297	2724	3035	3911	-
t	225-900*	-	-	-	-	-	-
NEGAVNS	1278	1582	2297	2724	3021	3874	5493
t	1	4	26	6	38	22	34

Table 5. Quality (makespan) and time (s) needed by several sequential hybrid algorithms applied to some of Taillard's (1993) Fm|prmu|Cmax problem instances

The results compared in (table 5) come from the following methods: TAI – Taillard (1993), NS – Nowicki & Smutnicki (1996), LH – Ladhari & Haouari (2005), BLS – Haouari & Ladhari (2003), HPSO – Kuo et al. (2008), BDS – Framinan & Pastor (2008) and NEGAVNS – Zobolas et al. (2008). Unfortunately there are instances that cannot be compared due to lack of experiments and in other hand, we don't have references about the time to reach the best solution with BDS algorithm.

Also, it has to be noticed that experiments have been carried out in different computers with different level of power so the time to reach the minimum cannot be easily compared.

2.2.5. 2.1.1. Parallel algorithms

There are, as in sequential methods, very few references reporting the use of parallel hybrid methods that combine exact and approximate methods applied to the permutation flowshop or similar problems (table 5).

		Exact Method
		B&B
Approx. method	GA/EA	Pessan et al. (2008) Denzinger & Offerman (1999) Cotta et al. (1995)
	SA	Nwana et al. (2005)

Table 5. Parallel hybrid methods combining approximate and exact methods

Pessan et al. (2008) combine branch and bound with a genetic algorithm and apply it to scheduling with restart problems. Some of the methods

We have found several references to the use of agents systems or teams of agents. These methods can employ a different algorithm in each of the agents and build a parallel hybrid method. Puchinger & Raidl (2005) refers this architecture. The architecture is known as teams of agents, Asynchronous Teams (A-Teams) as described by Talukdar et al. (1998), by Talukdar (1998) and by Talukdar (1992) or TEams for Cooperative Heterogeneous Search (TECHS) as presented by Denzinger & Offerman (1999).

3. Future work

We are working now in a parallel hybrid algorithm combining B&B and CLM, see Ghosh and Sierksma (2002), based in BDS (Bound Driven Search), see Framinan and Pastor (2008). This algorithm will employ some of the ideas found in this survey regarding the collaboration and communication between processes.

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