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# A Mixed Integer Programming Model for Managing Rolling Stock in Railway Systems*

**Álvaro García-Sánchez, Miguel Ortega-Mier**

Dpto. de Ingeniería de Organización, Administración de Empresas y Estadística. Universidad Politécnica de Madrid. ETSII. Calle José Gutiérrez Abascal, 2. 28006 Madrid. alvaro.garcia@upm.es, miguel.ortega.mier@upm.es

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## 1. Introduction

A correct management of railway systems is a requirement so that they can offer satisfying levels of service quality without incurring in huge resource usage. Both effectiveness and efficiency are to be carefully addressed if railway is to maintain or achieve a predominant position among means of transport and, especially, among public ones.

For that to occur, a wide set of problems need to be addressed, ranging from strategic to operative ones. One of this problems consists in determining how convoys should be composed to that the attend the demand for a particular time horizon, without violating those physical constraints involved in composing and decomposing convoys and, preferably, with a lower usage of the resources involved. This problem is referred to as the Rolling Stock problem.

This paper presents a model for solving a Rolling Stock problem which corresponds to the problem in the Madrid regional railway.

In the literature there can be found many papers related to problems that appear in railway systems management, among which there are some major contributions to the Rolling Stock problem.

The first major contribution is that of Schrijver (1993), where the problem consists in determining the number of units that must compose every convoy to attend the demand. No much detail is included but very elemental constraints.

Peeters and Kroon (2003) and Folkman et al (2004) give much more detailed models, which represent the system under study in much more detail. Some of the assumptions are not valid for the system under study in this paper. For example, since these works deal with medium distance trips, stops are significantly larger and what may and not may be done during those stops is different. Besides, two different classes are considered which make no sense for our system. These models are highly complex and specific method need to be developed to solve them.

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The problem addressed in this paper lay somewhere in between those that are very simple and those that are highly complex. The model presented later is detailed enough to represent what is necessary for the system under study but not that much that general branch and bound techniques cannot solve it. In all, the model lies somewhere between operational and strategic models.

The rest of the paper is organized as follows. Following the problem is described. Afterwards, the integer linear formulation is presented. In the next section the case study for the region of Madrid is described. Finally, some computational results are offered and guidelines for further research are discussed.

## 2. Problem description

In this section, we first present how the system operates. Later, the decisions to be made are presented. Two different criteria are considered and briefly discussed to assess how good solutions are. Finally and graph representation of the problem is presented, which is very helpful to formulate the model for this problem.

### 2.1. System operation

The problem addressed in this paper refers to a railway line consisting of a set of stations connected with tracks. This problem consists in determining how trains are to be composed so that the system can operate meeting the demand optimizing some relevant measure of the system performance.

Let us define a *service* as a journey departing from an origin station at a certain instant and arriving at a destination station after some time spam which is done by a set of rolling stock material which does not change during the whole journey.

Trains are composed of *Transportation Units* (TUs). A TU is the minimum amount of rolling stock that can be assigned to a service. Besides services must be assigned to a integer number of TUs.

In general terms, the problem consists in determining how many TUs are assigned to every single service within a given timetable for a determined period of time.

Of all stations, from our point of view only those ones where trains can be modified are of our interest, since no decisions can be made regarding rolling stock in those stations where the train composition cannot be changed.

Generally, these stations where trains can be altered are also parking stations or depots. The capacity for these depots may vary over time. For example, when the day ends, the regular tracks can be used to store trains whereas this cannot be done while the system is operating.

As an extreme situation of the previous case, there may be stations that act as depots overnight but cannot store trains at all during the operative time. Accordingly, this stations cannot be used to alter the composition of trains either.

We will assume that depots contain the same number of UTs at the beginning and at the end of the considered horizon (whether it is a day or a week, for example).

One of the issues that conditions how trains are composed is the demand for every service. Although a service transits different stations and its stretches result in different level of occupation, the solely important information as to the demand is that stretch which a higher demand. If the rolling stock assignment can cope with the demand of that stretch, it will with the demand for the rest of stretches. Hereafter, the term arc demand will refer to the demand corresponding to the stretch of the considered service with the highest demand.

It is noteworthy that demand changes over time. For a particular day there exist rush hours. Besides, flows across the line may not be symmetric, in the sense that rush hours may not coincide in time for the two senses of the line. The may suggest the use of non-serviceable trains, which are trains that are programmed to move material among stations so that UTs are available when need. Moreover, these non-serviceable trains may be totally necessary to achieve a feasible program.

### 2.2. Decisions involved

For a timetable, this is, a set of services to be carried out, the problem consists in determining:

- the composition for trains, which is the number of UTs for every service, and
- the number and composition of non-serviceable trains among those proposed by the railway managers**.**

### 2.3. Bounty criteria

Two main criteria can be of interest.

- Number of UTs. If operating conditions change, it can be of interest to calculate the minimum amount of units that are necessary to meet all constraints and attend the demand. The most desirable situation is needing the least number of UTs to satisfy the schedule.
- Number of unit·km. In a situation where the number of units is not a variable over which we can decide, and the number is large enough to meet the demand, it is preferable to use as less trains as possible, since this means lower operating and maintenance costs.

### 2.4. Graph representation

The problem can be expressed in terms of a graph such as that in figure 1. This is a simplified graph for the sake of clarity which contains as many circles as stations where train composition can change or where UTs can be stored, if not at any time, at least during certain periods over the considered horizon.

In every circle, there are a set of nodes, which we will refer to as events, where something occurs in a particular station. These events are characterized by a station and an instant.

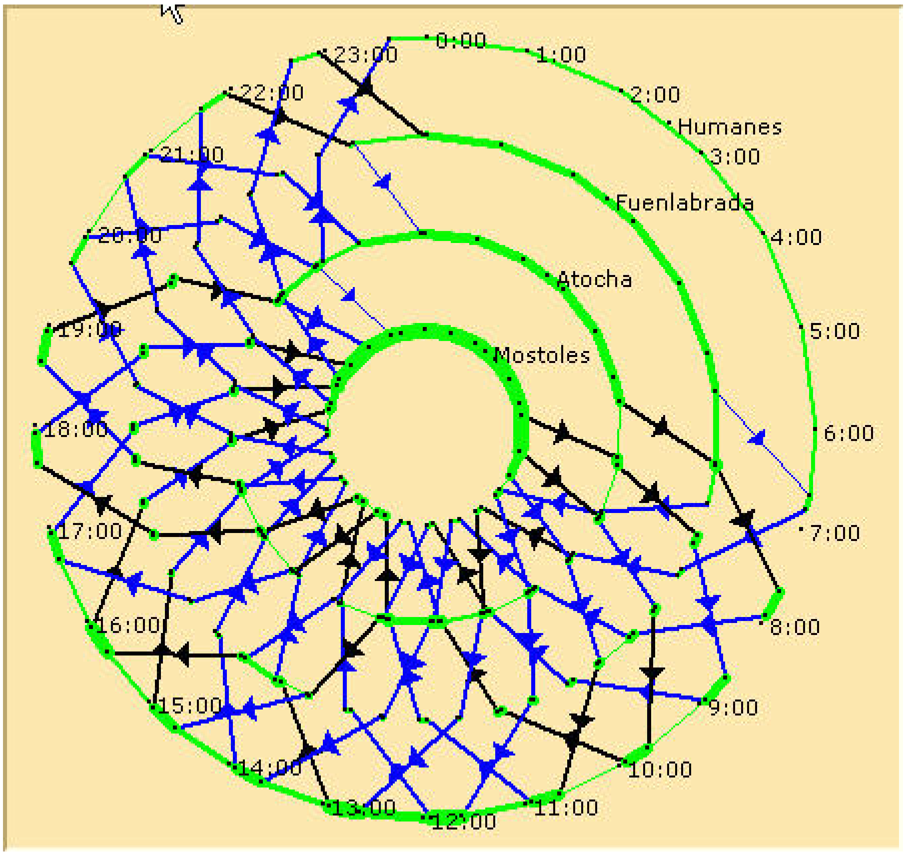


**Figure 1.** Figure with the circular graph

These events are connected by arcs, which represent two different things:

- Those arcs connecting events within the same circle represent those UTs that remain in that station during the time elapsed between the instants corresponding to those Events.

  There are some special arcs of these type, which are those connecting the last instant of the day for a station and the first instant of the day for the following day. These arcs will be helpful to compute the total number of units for a particular solution.

- Those arcs connecting events in different circles correspond to trains departing from a station at a particular instant and arriving to another station some time later.

As will be formulated in the following section, balance flow has to be maintained in every node (event). Besides, the circular nature of the graph automatically imposes the condition that the number of UTs in every station does not vary from the end of a day to the beginning of the following one.

## 3. Formulation

In this section, the IP model for the previous problem is presented. Sets are listed first, and afterwards, parameters and variables are defined over those sets. Finally the constraints and the two possible objective functions are given.

### 3.1. Sets

| | |
|---|---|
| $N$ | Set of stations |
| $T$ | Set of instants |

| | |
|---|---|
| $E$ | Set of events $E \subset N \times T$, where en event is defined by a station at which something happens at a particular instant |
| $A^P$ | Set of arcs corresponding to serviceable trains (transporting passengers), which depart from a event $e \in E$ and arrive at an event $e' \in E$, where those two events correspond to different stations and to different instants. |
| $A^{NM}$ | Set of non-movement arcs corresponding to those trains which are stored in a station between two events $e \in E$ and $e' \in E$, where these two events share the same stations but correspond to two consecutive events for that station. |
| $A^E$ | Set of arcs for empty trains (not transporting passengers), which depart from a event $e \in E$ and arrive at an event $e' \in E$, where those two events correspond to different stations and to different instants. |
| $A$ | Set of arcs, $A = A^P \cup A^{NM} \cup A^E$ |
| $A^0$ | Set of night arcs, which are those arcs linking the last event of the day for station $n \in N$ with the first event of the day for the very same station. Note that $A^0 \subset A^{NM}$, this is, $A^0$ contains those non-movement arcs corresponding to the transition from one day to the following one. |
| $\delta^+(e)$ | Set of arcs which end at event $e \in E$ |
| $\delta^-(e)$ | Set of arcs which start at event $e \in E$ |

### 3.2. Parameters

| | |
|---|---|
| $DEM(a)$ | Maximum demand (number of passengers) of arc $a \in A^P$ |
| $CAP_{UT}$ | Capacity (number of passengers) of a UT |
| $CAP_{ST}(a)$ | Capacity (number of UTs) that can be stored in the station to which corresponds the non-movement arc $a \in A^{NM}$. (Some stations have different capacities at different points in time). |
| $MAX_P(a)$ | Maximum number of units that can be used to compose serviceable trains for arc $a \in A^E$ |
| $MAX_{ET}(a)$ | Maximum number of units that can be used to compose non-serviceable trains for arc $a \in A^E$ |
| $D(a)$ | Physical distance corresponding to arc $a \in A$. This distance is that between the two stations corresponding to this arc. Not that $D(a) = 0 \quad \forall a \in A^{NM}$ |

### 3.3. Variables

| | |
|---|---|
| $f(a)$ | Number of UTs for arc $a \in A$. Depending of the type of arc, that may represent the UTs for a service the number of UTs for an empty train of the number of trains that remain in a station. $f(a) \in \mathbb{Z}^+$ |

### 3.4. Constraints

Services cannot be composed with more than two UTs:

$$f(a) \le MAX_P(a) \quad \forall a \in A^P \tag{1}$$

The maximum demand has to met for every serviceable arc:

$$f(a) \cdot CAP_{UT} \ge DEM(a) \quad \forall a \in A^P \tag{2}$$

Trains stored in stations cannot be larger that the capacity for each station, which may vary over time:

$$f(a) \ge CAP_{ST}(a) \quad \forall a \in A^{NM} \tag{3}$$

Non-serviceable (empty) trains cannot be composed with more than a certain number of UTs:

$$f(a) \ge MAX_{ET}(a) \quad \forall a \in A^E \tag{4}$$

UTs balance must be maintained in every node, which means that the number of incoming, outgoing and remaining trains must add up:

$$\sum_{a \in \delta^+(e)} f(a) \ge \sum_{a \in \delta^-(e)} f(a) \quad \forall e \in E \tag{5}$$

### 3.5. Objective functions

As discussed earlier, two possible objective functions can be considered, which refer to the pool of UTs and

#### 9.1.1 Number of units

$$\min \sum_{a \in A^0} f(a) \tag{6}$$

Since the sum in equation (6) is over the night arcs, it is an easy calculation to compute the total number of UTs: during the night trains are only stored in depots (described by $f(a),\ a \in A^0$) and there are no UTs elsewhere.

#### 9.1.2 Number of km·unit

$$\min \sum_{a \in A^P \cup A^E} D(a) \cdot f(a) \tag{7}$$

In this case the sum is over those arcs corresponding to movements of UTs, and each term in the sum corresponds to the physical distance of that arc times the number of UTs.

## 4. Case study

The previous model can be applied to any system or subsystem which operates as described. This may be the case of many railway systems within the Spanish context.

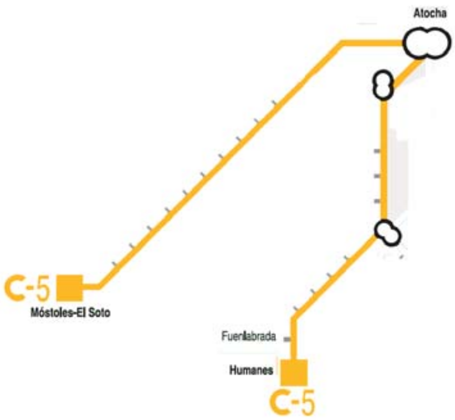


**Figure 1.** C5 line structure

In particular, this model has been applied to a line from the railway system of the region of Madrid, the C5 line. Figure 1 offers a simplified sketch for that line.

This line consist of 17 stations. Among them those that are relevant for the model are those whose names can be read:

– Fuenlabrada, Humanes y Móstoles are three stations where composition of trains can be changed along the day. Besides, they can store a certain amount of UTs during the whole day, which can be increased during the night.

– Atocha, where UTs can be stored only overnight. This is one of the busiest stations of the whole system. Many tracks can be found at this station but none can be used for changing the composition of trains. Nevertheless, since its activity completely stops during the night, some UTs can be stores using those tracks which are used for regular operation during the day.

In this particular case, the parameter which determines the maximum amount of UTs that for a train $MAX_P(a) = 2$, single or double

Trains can change in three station: Fuenlabrada, Humanes y Móstoles

More detailed data cannot be presented due to confidentiality agreements between the authors and Renfe.

## 5. Implementation and computational results

The model for the C5 line was built using AIMMS 3.8 (Bisschop and Roelofs, 1999) and solved using CPLEX 10.1 (ILOG, 2006) on an Intel Core 2 Duo 6320 1.86 GHz 2Gb RAM running under Windows XP.

The constraints and variables are 1283 and 978, respectively. All the variables are integer. The solving times are hundreds of seconds, which enables the possibility of quickly test several alternatives considering different alternatives

## 6. Conclusions and further research

In this paper, we have presented a tactical model for addressing the rolling stock problem which has been applied to a real system in a line of the railway system of the region of Madrid.

This model can be enhanced by slightly increasing its level of detail by including shunting operations.

Other source of further development would consist in considering several lines jointly, which will pose some new characteristics that will require adequate modelling.

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