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Specialization versus diversification: non-homogeneity in Data Envelopment Analysis

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1. Introduction

Although Data Envelopment Analysis has gone a long way since it was formulated by Farrell (1957), there are still some problems whose solution remains unsatisfactory, one of them being homogeneity. Homogeneity is a key assumption within Data Envelopment Analysis (DEA) where all Decision Making Units (DMUs) are required to conform to three rules: first, they should undertake the same processes; second, they should use the same inputs to produce the same outputs; lastly, it is required that they operate within the same environment: Dyson et al. (2001), and Haas and Murphy (2003).

In practice, homogeneity is seldom present. It is common for data sets to contain non-homogeneous units. For instance, we may be interested in assessing the efficiency of bank branches. Homogeneity would require all of them to engage in the same activities, but large branches will carry out most banking activities, whilst smaller branches may only engage in some of them. Standard use of DEA would prevent direct efficiency comparisons between small and large bank branches. Two ways of proceeding have been followed under these circumstances, either to base the analysis on a limited number of activities shared by all DMUs; or study only a limited number of DMUs that engage in exactly the same activities. Both solutions are clearly unsatisfactory.

Many attempts have been made in the past to study efficiency when DMUs were not homogeneous. Sarrico and Dyson (2000) tried to compare the efficiency of departments at Warwick University. They found that not all departments shared the same inputs: Science departments required laboratories and equipment while a Humanities department did not. They overcame this problem by running a DEA model for each department against external competitors in the same area, a very different matter from comparing departments within a university, the objective they had originally set to achieve. Another example is provided by Athanassopoulos and Thanassoulis (1991) who studied efficiency in the brewery industry, and tried to overcome the problem of non-homogeneity by grouping breweries into those that had passing trade and those that did not. The breweries were then analysis separately and gained an efficiency score within their group but, again, this is not an ideal solution.

Non-homogeneity was also encountered by Ray (1991) when studying public schools; by Fizel and Nunnikhoven (1992) in the area of nursing homes; by Sexton et al. (1994) in pupil transport; and by Zenios et al. (1999), and Soteriou and Zenios (1999) in banking. All of them devised ad hoc rules in order to deal with this problem.

A popular way to deal with the homogeneity problem is by using a two-step procedure. In the first step, DEA scores are calculated, and in the second step, these scores are regressed against possible causes of non-homogeneity that had not been included in the original formulation. Examples are: Ray (1991), Sexton et al. (1994), Fizel and Nunnikhoven (1992), Mancebon and Mar-Molinero (2000), and Hass and Murphy (2003).

Here we propose a new approach to a common form of non-homogeneity, the one encountered when not all DMUs share the same inputs and/or outputs. This model can also be used to address the question of whether it is better to diversify or to specialize.

The rationale of this new model will be presented and the model formulated. Equations will be given for the envelopment and the ratio form of the model. The model will be demonstrated with some data from Beasley (1995). The paper will end with a concluding section.

2. The model

For clarity of presentation, it will be assumed that we are trying to assess efficiency in university level institutions. There will be three types of university institutions in the assumed data set: those, such as standard universities, that engage in both teaching (T) and research (R); those that engage in teaching but not in research; and those, such as research institutes, that engage in research but not in teaching. We would like to study the efficiency with which these institutions conduct the R and the T functions by using the complete, non-homogeneous, data set. In the case of institutions that perform both the T and the R function, this will involve estimating a DEA score for the T activity and a DEA score for the R activity.

This problem has another interpretation. We are, in fact, asking the question of whether it is better to specialize (leave the T function to T only institutions, and the R function to R only institutions), or to diversify (conduct both activities jointly). We have not found any other attempt in the DEA literature of addressing the diversification versus specialization issue.

As in any DEA problem, there are two formulations for the model: the ratio form, and the envelopment form. We think that the philosophy of the modeling procedure is better understood within a ratio formulation, although we will mathematically formulate the equations for the envelopment form. The ratio form of the model will also be given for completeness.

The standard DEA model, in the ratio formulation, is often interpreted as follows.

"Take the DMU whose efficiency we wish to assess. Define efficiency as the ratio of weighted outputs to weighted inputs. The DMU under observation is allowed to choose the weights to be used in this ratio so that its efficiency is maximized, but once such weights are chosen they are applied to study the efficiency of the remaining DMUs in the data set. If, using the same weights as the DMU under observation, no other DMU achieves a higher level of efficiency, the DMU under observation is efficient. If, using the same weights as the DMU under observation, other DMUs achieve higher efficiencies, the DMU under observation is inefficient". Of course, this story needs to be completed with the conditions that the weights need to be strictly positive, and that efficiencies are positive numbers between zero and one.

The problem of estimating the efficiencies of activities that are jointly performed was studied, under constant returns to scale, by Beasley (1995), Mar-Molinero (1996), and Mar-Molinero and Tsai (1997); and, under variable returns to scale, by Tsai and Mar-Molinero (2002). This model has been applied to the UK health service by Tsai and Mar-Molinero (2002), to police forces by Diez-Ticio and Mancebon (2002), to the study of education costs by Salerno (2006), and to bus services in Taiwan by Yu (2007).

Under the joint efficiency model, some inputs are allocated only to the T activity, some inputs are shared between the T and R activities, and some inputs are allocated only to the R activities. In the same way, some outputs can be attributed to the T activity, some outputs reflect the effort devoted to the T or the R activity, and some outputs depend only on the R activity. The DMU under observation has to decide how to allocate shared inputs to the R or to the T activities, and how much effort to devote to produce outputs from the T or the R activities. This it does by taking into account the importance attached to the T activity, the importance attached to the R activity, and the desire to be seen to be operating as efficiently as possible under both activities when compared with other DMUs.

The rationale of joint efficiency DEA algorithm is based on the same philosophy as before: once the DMU under observation has decided how to allocate shared inputs, and how to attribute shared outputs, this split is applied to all other DMUs and efficiency calculations take place as usual. Efficiency calculations can take place as usual because, once the split of shared inputs and the split of shared outputs have been decided, we face a standard DEA problem for the T activity, and a standard DEA problem for the R activity.

The ability to split the joint problem into a T problem and a R problem allows us to incorporate the T only institutions, which will be compared with the T part of the institutions that engage in both T and R activities; and R only institutions, which will be compared with the R part of the institutions that engage in both T and R.

We now give the mathematical formulation for the complete model. This requires introducing notation.

Let there be I outputs, and J inputs. Let there be S DMUs that engage in both T and R, P DMUs that only engage in T, and Z DMUs that only engage in R. Let the DMU whose T and R efficiencies we wish to calculate be indexed as k. We will be calculating efficiencies under the output oriented formulation with variable returns to scale. The model can be modified in order to accommodate other formulations.

y_{is}^{T} is the amount of output i associated only with the T activity for DMU s.

y_{is}^{R} is the amount of output i associated only with the R activity for DMU s.

y_{is}^{TR} is the amount of output i associated with both the T and the R activities for DMU s, of which a proportion βi can be attributed to the T activity and a proportion $1- \beta i$ can be attributed to the R activity.

x_{js}^{T} is the amount of input j allocated only to the T activity of DMU s.

x_{js}^{R} is the amount of input j allocated only to the R activity of DMU s.

x_{js}^{TR} is the amount of input j allocated in part to the T activity and in part to the R activity of DMU s. A proportion μj is allocated to the T activity, and a proportion $1-\mu j$ is allocated to the R activity.

λ_s^{T} is the DEA multiplier for the T activity of DMU s.

λ_s^{R} is the DEA multiplier for the R activity of DMU s.

w_k^{T} is the inverse of the T efficiency for the DMU under observation, k.

w_k^{R} is the inverse of the R efficiency for the DMU under observation, k.

The importance attached to the T activity and to the R activity is captured by the weights θ^{T} and θ^{R}. These weights are determined outside the model and reflect the priorities of the decision maker. It is customary to choose them so that they add up to unity.

Finally, let the overall efficiency of DMU k be measured by e_k.

We are now in a position to write down the equations for the model.

The objective is to maximize the output obtained from the resources used by DMU k.

$$Max \frac{1}{e_k} = \theta^T w_k^T + \theta^R w_k^R$$

We will now turn to the constraints and we will start with the inputs. Institutions that engage in both the T and the R activities may use some inputs that are specific to the T function, while institutions that only engage in the T activity (and do not engage at all in the R activity) may use the same inputs. This will produce equations of the form:

$$\sum_{s=1}^{S} \lambda_s^T x_{js}^T + \sum_{p=1}^{P} \lambda_p^T x_{jp}^T \leq x_{jk}^T$$

There will be a similar equation for each of the inputs that are allocated only to the R activity by institutions that engage in both T and R. These inputs could also be allocated to R if assigned to R only institutions.

$$\sum_{s=1}^{S} \lambda_s^R x_{js}^R + \sum_{z=1}^{Z} \lambda_z^R x_{jz}^R \leq x_{jk}^R$$

In line with the formulation given by Mar-Molinero (1996), inputs that can be allocated in part to the T activity and in part to the R activity will produce only one equation. These inputs could also be allocated to T only institutions, and used only for T purposes, or allocated to R only institutions and used for R only purposes. This produces equations of the type:

$$\sum_{s=1}^{S} \lambda_s^T \mu_j x_{js}^{TR} + \sum_{s=1}^{S} \lambda_s^R (1 - \mu_j) x_{js}^{TR} + \sum_{p=1}^{P} \lambda_p^T x_{jp}^{TR} + \sum_{z=1}^{Z} \lambda_z^R x_{jz}^{TR} \leq x_{jk}^{TR}$$

We now turn to the constraints associated with outputs. Outputs that are only the result of the T activity produce

$$\sum_{s=1}^{S} \lambda_s^T y_{is}^T + \sum_{p=1}^{P} \lambda_p^T y_{ip}^T \geq w^T y_{ik}^T$$

The corresponding equation for R only outputs is:

$$\sum_{s=1}^{S} \lambda_s^R y_{is}^R + \sum_{z=1}^{Z} \lambda_z^R y_{iz}^R \geq w^R y_{ik}^R$$

And the equation for outputs that are the result of both T and R is

$$\sum_{s=1}^{S} \lambda_s^T \beta_i y_{is}^{TR} + \sum_{s=1}^{S} \lambda_s^R (1 - \beta_i) y_{is}^{TR} + \sum_{p=1}^{P} \lambda_p^T y_{ip}^{TR} + \sum_{z=1}^{Z} \lambda_z^R y_{iz}^{TR} \geq w^T \beta_i y_{ik}^{TR} + w^R (1 - \beta_i) y_{ik}^{TR}$$

Under variable returns to scale, two additional constraints are required, one for the T activity

$$\sum_{s=1}^{S} \lambda_s^T + \sum_{p=1}^{P} \lambda_p^T = 1$$

and one for the R activity:

$$\sum_{s=1}^{S} \lambda_s^R + \sum_{z=1}^{Z} \lambda_z^R = 1$$

It is possible for this formulation to produce outputs without any inputs by, for example, setting the value of μ_j either to zero or to one. To avoid the possibility of producing outputs without inputs we need some further constraints:

$$m_j \le \mu_j \le M_j$$

$$b_i \le \beta_i \le B_i$$

The formulation is completed with the usual limiting conditions that require that all unknowns be positive. The efficiency factors w^T and w^R are required to be greater than one, but the model ensures this automatically.

See Figures 1, 2, and 3 for a schematic representation of the structure of DMUs that engage in both T and R, and DMUs that engage only in T.

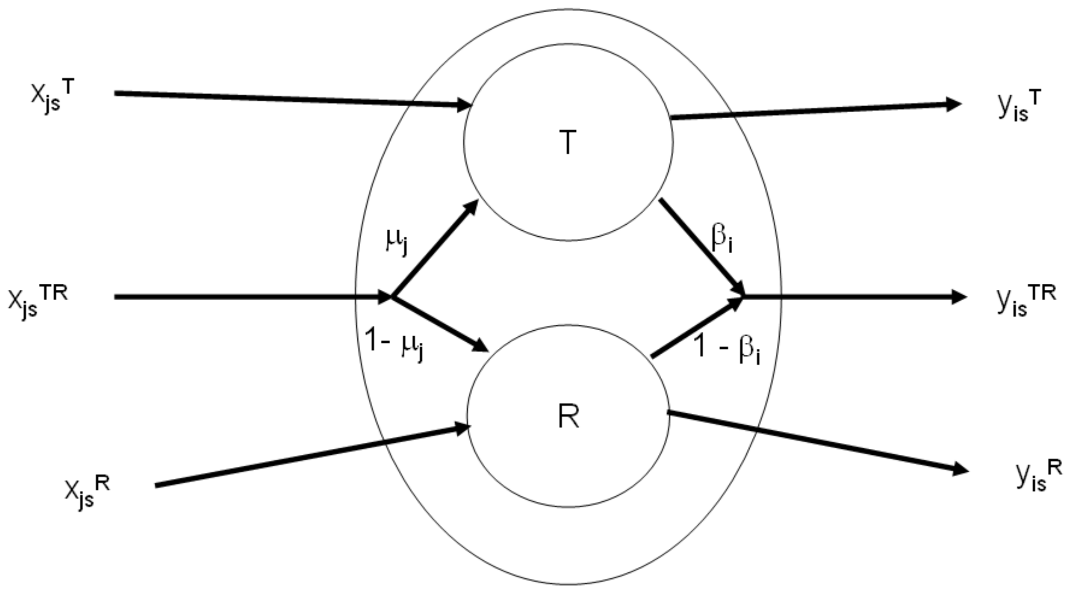

Figure 1.- Structure of a DMU that engages jointly in T and R

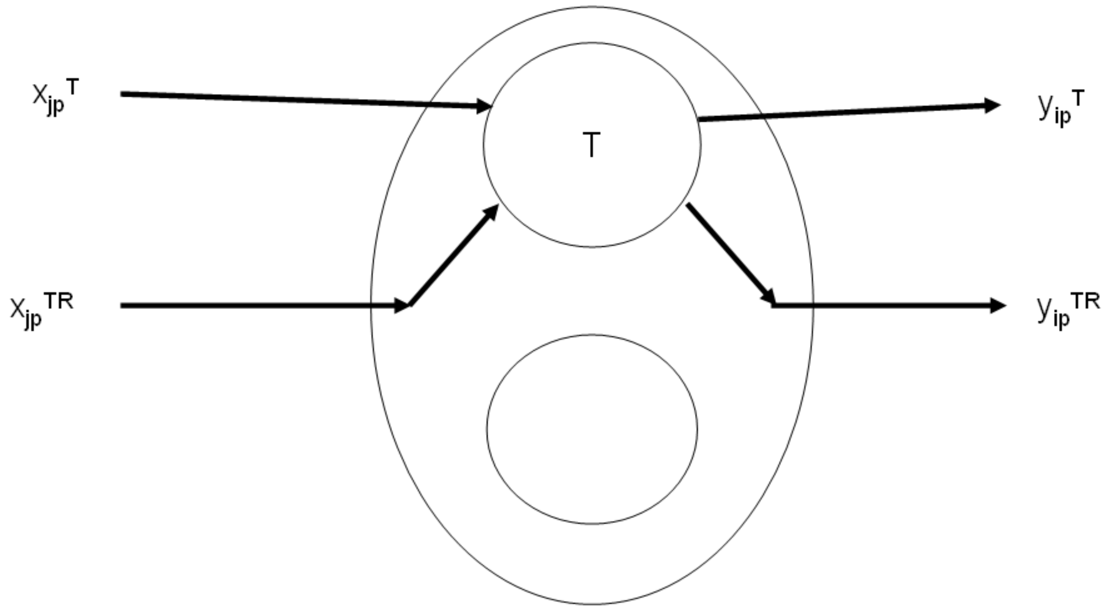

Figure 2.- Structure of a DMU that engages only in T

2.1. Example

The model will be demonstrated on some data for university physics departments provided by Beasley (1995). Each physics department is a DMU. In Beasley's paper there are only two activities: Teaching and Research. In this paper we will consider three activities: undergraduate teaching (UT), postgraduate teaching (PT), and research (R). For the purposes of this paper we will consider all three activities to be of equal importance; i.e., the weights θ^{UT} , θ^{PT} and θ^{R} were each set to one third in the objective function when all three activities were present. When a department did not engage in PT, the weight θ^{PT} did not appear in the objective function, and the other two weights were set to one half each.

Beasley's data set is not homogeneous, as there are many DMUs that do not engage in PT. It is exactly the situation that can be modeled with the equations presented here.

We will use the same inputs as Beasley: equipment expenditure, and general expenditure. DMUs will attempt to allocate these expenditures between the activities. A lower limit of 0.1 and an upper limit of 0.9 were set for the proportions in which these shared inputs can be divided.

The number of undergraduate students is the output of the UT activity. The number of postgraduate students is the output of the PT activity. Given the way in which universities work in the UK, this is a reasonable distinction to make, as engaging in postgraduate teaching is a departmental decision, while undergraduate teaching is the result of a national negotiation between the funding bodies and the universities. As in Beasley, the research activity generates research students and research income. Beasley uses a third output for the R activity, research rating, but this is not used in this paper.

The model was estimated with specially written software using the package LINGO. The results are shown in Table 1. It is seen that only two departments are 100% efficient, and that these are departments that do not carry out the PT activity. There are also a number of departments that are efficient in certain activities, but not overall. Whilst the model is not the exact replica of Beasley's (1995), it does show similar results within the overall efficiencies.

Physics department	Overall Efficiency	Undergraduate Efficiency	Research Efficiency	Postgraduate Efficiency
Aberdeen	6.61%	19.77%	30.10%	2.70%
Aberystwyth	10.41%	15.63%	25.41%	5.41%
Bangor	15.83%	14.14%	17.97%	N/A
Bath	42.78%	33.88%	58.01%	N/A
Birmingham	46.72%	73.05%	36.49%	43.24%
Bradford	7.58%	56.64%	3.87%	8.33%
Bristol	47.24%	52.76%	42.77%	N/A
Brunel	28.46%	20.35%	100.00%	21.62%
Cambridge	25.44%	64.71%	100.00%	10.81%
Cardiff	26.57%	29.03%	24.49%	N/A
Dundee	28.30%	29.58%	18.05%	59.51%
Durham	42.56%	37.32%	49.53%	N/A
East Anglia	24.36%	13.87%	100.00%	N/A
Edinburgh	7.03%	58.94%	25.14%	2.70%
Essex	28.11%	17.19%	37.34%	45.95%
Exeter	30.59%	47.39%	22.59%	N/A
Glasgow	13.45%	78.55%	39.44%	5.41%
Herriot-Watt	6.78%	32.48%	23.80%	2.70%
Hull	33.69%	25.55%	49.45%	N/A
Keele	27.99%	24.74%	32.21%	N/A
Kent	34.37%	29.61%	40.98%	N/A
Lancaster	30.15%	24.82%	45.03%	27.03%
Leeds	32.17%	31.07%	33.35%	N/A
Leicester	38.18%	31.02%	49.65%	N/A
Liverpool	29.76%	32.35%	23.01%	37.84%
London Birkbeck	13.98%	5.15%	100.00%	97.30%
London Imperial	76.87%	99.09%	77.84%	62.16%
London KQC	44.53%	36.40%	57.34%	N/A

London Q. Mary	20.50%	39.95%	20.28%	13.89%
London R. Hol Bed	21.62%	33.40%	61.26%	10.81%
London Univ. Coll.	7.55%	40.51%	100.00%	2.76%
Loughborough	36.89%	28.54%	52.17%	N/A
Manchester	71.21%	64.89%	74.05%	75.68%
Newcastle	45.03%	35.65%	100.00%	35.00%
Nottingham	46.93%	46.51%	47.36%	N/A
Oxford	100.00%	100.00%	100.00%	N/A
Reading	27.83%	20.14%	22.57%	72.22%
Salford	37.84%	29.39%	41.52%	47.22%
Sheffield	7.12%	61.27%	28.44%	2.70%
Southampton	39.67%	41.69%	32.24%	48.51%
St. Andrews	44.85%	29.55%	81.50%	48.13%
Stirling	100.00%	100.00%	100.00%	N/A
Strathclyde	42.15%	57.06%	33.42%	N/A
Surrey	44.60%	21.87%	86.55%	100.00%
Sussex	51.26%	37.77%	64.54%	60.39%
Swansea	22.17%	19.87%	31.11%	18.92%
UMIST	33.28%	23.71%	55.79%	N/A
Warwick	42.75%	39.89%	46.05%	N/A
York	15.77%	27.76%	32.41%	8.11%

3. Conclusion

This paper has explored a new way of dealing with one type of non-homogeneity in DEA, the one that manifests itself when some Decision Making Units do not carry out the same activities as others, and they do not share some inputs or outputs.

Direct comparisons of non-homogeneous units can now be made without the need for unnecessary assumptions within the model. This paper has looked at how the models formulation works and shows the dual calculations. Beasley's (1995) data has been used as an illustration of how the model runs.

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