

Using Greedy Clustering Method to Solve Capacitated Location-Routing Problem

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Abstract

Location-Routing Problem (LRP) is related to logistics system of supply chain. In LRP, aims are facility location problem and vehicle routing problem that are considered simultaneously. As shown in recent researches, the costs in distribution systems may be excessive if routes are ignored when locating depots. In this paper, a heuristic method in four phases is developed to solve Capacitated Location-Routing Problem (CLRP). In phase 1, the customers are clustered according to greedy search method. Second phase, chooses the proper depots among candidates to be established. Third phase, allocates the clusters to depots. In final phase, Ant Colony System (ACS) is used for routing among depots and customers. The experimental results show the efficiency of our approach.

1. Introducción

Location-routing problem is usually related to distribution systems and supporting supply chain. This problem is so important and valuable for supply chains that delivery goods to customers on time is of primary objective of every manager in a company. LRP has many applications in various fields such as: health, military and communications. The main interest about LRP is when customers demand is permanent (like collecting garbage) or when the depots are temporary (like demand in different seasons) (see Prodhon, 2007).

For the first time, Maranzana (1964, p.216) points out that: “the location of factories, warehouses and supply points in general ... is often influenced by transport costs.” (Researchers consider this paper as the first publication on the LRP). Rand (1976), and Salhi and Rand (1989) showed that separating depot location problem and vehicle routing problem may face some suboptimal answers.

Nagy and Salhi (2007) presented a review of early works on LRP and summarizes the different types of formulations, solution methods and problems with non-standard hierarchical structure of work published prior to 2007. Min et al. 1998 mentioned in their survey, most early published papers considered either capacitated routes or capacitated depots, but not both (see Laporte et al. 1988; Chien 1993; Srivastava 1993).

In CLRP, the situation is that demand and supply points are given on the plane. Customers' demands are given according to previous patterns and data. The capacity and ability of keeping the goods are determined from candidate depot centers. The vehicles transfer the goods with specific capacity and type from depots to the customers and then return to related centers. The aim is to determine optimum depot centers and also to provide and distribute the

goods from depots to the customers. These two aims are together main goals of the problem. Because of capacitated location-routing problem is of NP-hard and its solution's times is long (see Srivastava, 1986), mainly heuristic or metaheuristic methods are used to solve it. Presented heuristic methods are usually divided into four groups: Sequential Method, Clustering Method, Iterative Method and Hierarchical Method.

Tuzun and Burke (1999) introduced a two-phase approach that coordinates two Tabu Search (TS) mechanisms: one searching for a good facility configuration, and the other a good routing that corresponds to that configuration. Albareda-Sambola et al. (2005) developed a two-phase Tabu Search (TS) heuristic for the LRP with one single route per capacitated open depot. The two phases consist of an improvement that optimizes the routes and a Permutation that modifies the set of open depots. To solve the CLRP, a cluster analysis based on sequential heuristic that uses simple procedures was presented by Barreto et al. (2007). Moreover, four grouping techniques (hierarchical and non hierarchical) and six proximity measures were used to obtain several versions of the heuristic.

In this paper, a heuristic method is presented to solve CLRP which includes four phases. firstly, the customers are clustered according to greedy search method. Phase 2, choose the proper depots among candidates to be established. Phase 3, allocates the clusters to depots. In phase 4, ant colony system is applied for routing among depots and customers.

The remainder of this paper is organized as follows: Section 2 describes definition of capacitated location-routing problem, phases of heuristic methods with graphic figures. In the section 3, the details of each phase are explained and its algorithms are presented. section 4, shows the computational results. Finally, in section 5, the conclusion has been presented.

2. Problem definition and description of heuristic method

Capacitated location-routing problem is about delivering the goods from depot centers to customers. Hypotheses and aims of the problem are as follows:

Hypotheses:

- Demand and supply points are given on the plane.
- Each customer's demands and the capacity of keeping the goods in depots are specific.
- The capacity of the vehicles is specific.
- The vehicles are homogeneous and refer to the depot after transferring goods.
- Each customer must receive service just from one depot.

Aims:

- Determining optimum depot centers.
- Establishing tour of vehicle between depot and customers.

Total cost of locating depots (fixed cost of depots) and routing are measuring criteria which must be minimized. This paper presents a method for solving CLRP which use minimum number of vehicles (minimum number of clusters) and minimum number of depots. This can be considered as the aim of LRP.

The main body of presented heuristic method determines clusters of customer. In phase 1, to cluster customers, greedy search method is used to find the nearest city to existing city (as form tour in travelling salesman problem). To form clusters of customer, total customers' demands of each cluster must be less than or equal to the vehicle capacity (Fig. 1.a).

In phase 2, each cluster's gravity center (gravity center of the customers of each cluster) is calculated. Cluster's gravity center is used as the representative of the cluster to choose depot or depots. To choose depots for opening, facility location problem among candidate depots

and gravity center are solved. In this phase, the minimum number of depots are opened to

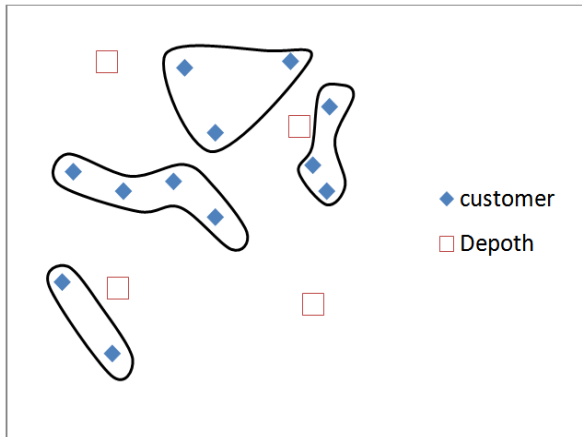


Figure 1.a. Clustering customers

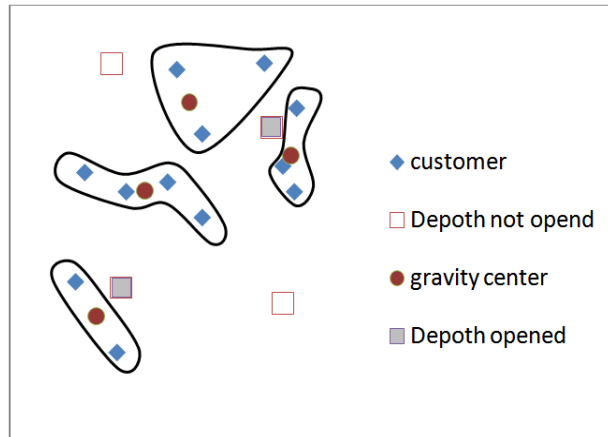


Figure 1.b. Determining gravity centers of clusters and choosing proper depots

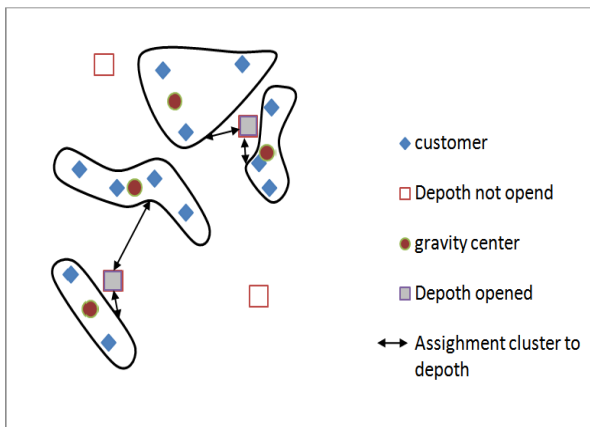


Figure 1.c. Allocating clusters to depots

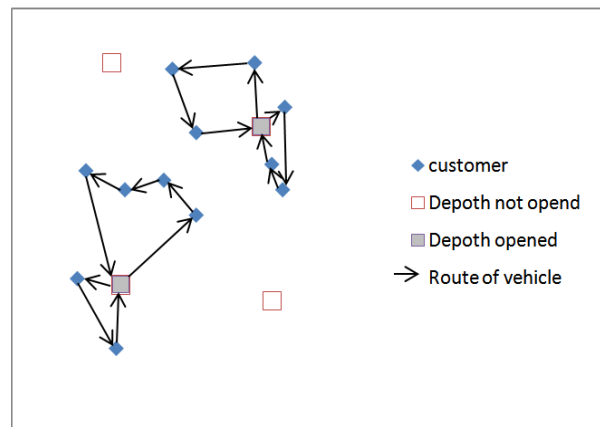


Figure 1.c. Forming tour between depots and customers

3. Details and algorithm of heuristic method

3.1 Clustering customers

In phase 1, customers are categorized in clusters. Each vehicle has to service one cluster. Therefore, total customers' demands of each cluster must be less than or equal to the capacity of the vehicle. To cluster customers, greedy search method has been used to choose the city in Traveling Salesman Problem (TSP).

Method of clustering is that one customer is chosen randomly and enters the cluster. To enter the second customer, it should have the closest gap with the first one. Then the demands of the first and second customer are calculated and if being less than the capacity of the vehicle, it enters the cluster and then among the remained customers (who does not have any cluster), one who has the closest gap with the second customer is chosen. If its demand and total customers' demands are less than the capacity of the vehicle, it enters the cluster. This process continues until all customers are clustered. Adding customers to each cluster is stopped if:

(1) Total customers' demands are more than the capacity of the vehicle because of the close gap of the last customer of the cluster. In this condition, the capacity of the vehicle is deducted from total customers' demands. The acquired amount is the remaining capacity of the cluster through which a list of customers who have demands in this boundary and are able to enter the cluster, are achieved. The reason of providing the list is that we can use maximum capacity of the vehicle. So this heuristic method uses minimum number of vehicles (or minimum number of clusters). According to this list, the customer who has the closest gap with the last one enters the cluster. This process continues until the customers can't enter the cluster because of filling the capacity of vehicle or because of the following reason.

(2) When members number of each cluster reaches to specific number, no one can enter the cluster while the cluster can accept the next customer. This condition has been used and causes the concentration of customers not to be more than other clusters and affects choosing depot in the next phase and final solution. Determining the maximum number of members of clusters is according to trial and error method.

3.2. Choosing depots

After forming the cluster in phase I, the gravity center of each cluster is calculated based on E.q (1) to be used as the representative of each cluster to choose proper depots among different candidates:

$$(X_{(I)}, Y_{(I)}) = \left(\frac{\sum_{i=1}^{n_I} x_i}{n_I}, \frac{\sum_{i=1}^{n_I} y_i}{n_I} \right) \quad (1)$$

Parameter definition:

$(X_{(I)}, Y_{(I)})$: Gravity center of I^{th} cluster

i : i^{th} customer of cluster I

n_I : Number of customers in I^{th} cluster

(x_i, y_i) : Coordinates of i^{th} customer

By defining the gravity center of each depot, we can start choosing depots among the candidates. Using single facility location problem (SFLP), the depot is chosen. In SFLP, a depot is chosen which has the minimum sum of distances with gravity centers. The distances are calculated as Euclidean distance. SFLP is in E.q (2).

In E.q (2), (x^*, y^*) is coordinates of selected depot among candidates :

$$(x^*, y^*) : \text{Min } w_j = \sum_{i=1}^m [(x_j - a_i)^2 + (y_j - b_i)^2]^{1/2} \quad \text{for } j = 1, \dots, n \quad (2)$$

Parameter definition:

w_j : Cost of opening of j^{th} candidate depot

(x_j, y_j) : Depot coordinates of j^{th} candidate

(a_i, b_i) : Coordinates of gravity center of i^{th} cluster

m : Number of clusters

n : Number of candidate depots

If selected depots can cover the customers' demands, phase 2 will be stopped. Otherwise, through updating depots (deleting selected depot), facility location problem is again solved. Phase 2 stops when the last depot can cover all customers' demands. So, this method always establishes minimum number of depots.

3.3. Allocating cluster to depot

Third phase allocates the clusters to the depots. By allocating cluster to depot, the vehicle must move from that depot and service to customers' demands and again return to the depot. Each depot can service some clusters based on its capacity. In this phase, among the opened depots, the any cluster allocates to a depot which has the closest gap with its gravity center. Also, the depot accepts the clusters according to its capacity. When the capacity of the depot is full, allocating the cluster is stopped and another opened depot is used according to closeness of gravity center to the depot. This method continues until all clusters allocate to depots.

3.4. Routing

According to phase 3, there are some special clusters for each depot to be serviced. Vehicles (equal to allocated clusters) are also ready to start movement from depot and after servicing to customers return to the depot. Each vehicle is responsible for one cluster. To get the best tour of the vehicle, Traveling salesman problem is solved through ant colony system.

Ant colony system (ACS) is referred to ants' treatment to find food. The ants spread a material which is called pheromone and put it on their way so that other ants can pass. The pheromone of shorter route increases and therefore, more ants move from that way. You can study more about ACS in Dorigo et al. (1996). Heuristic parameters of ACS algorithm have been considered as $\alpha=1$, $\beta=5$ and $\rho=0.65$.

According to phases, steps of proposed heuristic method for solving the CLRP is represented as follow:

Phase 0: Entering data

Receive customers' coordinate and save it in list A . Receive coordinate of candidate depot and save it in list D . Receive maximum number of customers in each cluster and save in N . Save sum of customers' demands in Sum . Set *iteration* equal to zero and q equal to number of algorithm iterations.

Phase 1: clustering customers

- (a) Select randomly a customer in list A and enter it in cluster i . Remove this added customer from list A .
- (b) Save the new customer's demand of the cluster in de .
- (c) Calculate the distance among the remained customers of list A with the last customer of the cluster i (according to their entrance) based on Euclidean distance.
- (d) Choose the customer who is closer to the last customer of the cluster i and add its demand to de .
- (e) If de is less than the capacity of the vehicle and number of the customers of the cluster i is less than N , go to step $e.1$, otherwise deduct the demand of the last customer that added to cluster i from de and go to step $e.2$.
 - (e.1) Enter the selected customer to the cluster i and remove it from list A . go to step c .
 - (e.2) Deduct the capacity of the vehicle from de and save it in Re .

- (e.2.1) Choose the customers who have demands less than or equal to Re from list A and save them in list C .
- (e.2.2) If list C is empty, enter the cluster i in list M and go to step f , otherwise, go to step $e.2.3$.
- (e.2.3) From the list C , choose the customer which is closer (by Euclidean distance) to final customer of the cluster i and add his demand to de .
- (e.2.4) If de is more than the capacity of the vehicle or the number of the customers of the cluster i is more than N , go to step $e.2.5$, otherwise, go to step $e.2.6$.
- (e.2.5) Stop adding customer to the cluster i and enter cluster i to list M . Go to step f .
- (e.2.6) Enter the selected customer to the cluster i and remove it from list C and then go to step $e.2.3$.
- (f) If list A is empty, go to phase 2. Otherwise, $l \leftarrow l + 1$, $de=0$ and $C=0$, then go to step a .

Phase 2: Choosing depots

Set the *Location Cost* equal to zero.

- (a) For clusters of list M , save the gravity center of i inside $G(i)$.
- (b) According to Euclidean distance, calculate the sum of distances of each depot inside list D with gravity center of clusters $G(i)$. (Location problem)
- (c) Save the depot which has minimum sum of distances with gravity centers, inside list M' to be opened and remove it from list D . Add the fixed cost of opening the mentioned depot to *Location Cost*.
- (d) If *sum* is less than or equal to total capacity of depots inside list M' , go to phase 3, otherwise, go to step b .

Phase 3: Allocating cluster to depot

Set F equal to zero.

- (a) Choose the first opened depot from list M' and save it in O .
- (b) For clusters of list M , Calculate the distance of gravity centers with depot O .
- (c) Save the cluster which has the least distance with depot O in J and do the following:
- (c.1) Calculate total customers' demands inside the cluster J and add to F .
- (c.2) If F is less than the capacity of depot O , go to step $c.3$, otherwise, go to step e .
- (c.3) Add the depot coordinate to the cluster J for touring, remove the cluster J from list M and enter the list M'' . Go to step d .
- (d) If list M is empty, go to phase 4. Otherwise, go to step b .
- (e) Remove depot O from list M' and choose the next depot (according to their opening) from list M' and save inside O . Set F equal to zero and go to step b .

Phase 4: Routing

Set the *Tour Cost* equal to zero.

- (a) Choose a cluster from list M'' and solve *TSP* using *ACS* to find the best tours among depot and customers.

- (b) Add the ACS result to *Tour Cost* and remove the mentioned cluster from M^m .
- (c) If list M^m is empty, do $Location Cost + Tour Cost \rightarrow Total Cost$ and go to step *d*. otherwise go to step *a*.
- (d) If $Total Cost < Best Cost$ then replace $Best Cost = Total Cost$. Go to step *e*
- (e) Do $Iteration | 1 \rightarrow Iteration$. If $iteration = < q$ go to first phase, otherwise stop the algorithm.

4. Computational results

In this section, computational tests are carried out on 11 standard instances (CLRP) of the literature⁹. Table 1 summarizes the results of the computational experiments. To consider efficiency of the presented heuristic method, its results are compared to Barreto et al. (2007) and Marinakis and Marinaki (2008). results. The whole algorithm of heuristic method is coded in MATLAB 7.0.4 and a system with Intel(R) Core (TM) Duo CPU T2450 2.00 GHZ is used to perform the program.

In first column of table 1, some examples with number of customers and candidate depots are presented by researches. Second column shows the capacity of the vehicle. The best solutions for these problems are in column 3 and the results of proposed heuristic method are represented in forth column. Fifth column shows also the quality of the obtained solution as percent of deviation from best exiting solutions. It is calculated as $\frac{(f_{Heuristic} - f_{BKS}) \times 100}{f_{BKS}} \%$ that $f_{Heuristic}$ is the best solution cost obtained by the proposed heuristic method and f_{BKS} denotes the cost of the best known solution. In sixth column, number of required depots are given for each problem. The presented heuristic method establishes the minimum depots. The minimum number of depots can be calculated through $Num\ Of\ Dep = \left\lceil \frac{D}{R} \right\rceil$ in which R is the capacity of depot and D is the total demands of customers. $\lceil \cdot \rceil$ is smallest integer number of greater than $\frac{D}{R}$. Number of necessary vehicles and maximum members of clusters of each problem are observed in column 7. The proposed heuristic method uses minimum number of vehicles (number of clusters). Minimum number of vehicles can be calculated through $Num\ Of\ Veh = \left\lceil \frac{D}{r} \right\rceil$ in which r is the capacity of the vehicle. Column 8 shows the required average time to get the answer. Proposed heuristic method, has improved 4 answers of standard problems. The bold numbers in table 1 indicate the best deviation found. One answer which is marked with an asterisk, has reached the lower bound. 4 answers have not been changed and 3 problems had also worse answers. Our researches show that the kind of clustering in the presented heuristic method is more efficient for some problems and improves the answers. It is inefficient or ineffective for some others. On the whole, the quality and the time of answers of the method are effective and considerable.

Table 1. Computational results of heuristic method on test problems

CLRP instance	Veh. Cap.	BKS	Heuristic method	Quality (%)	Num. of dep.	Num. of veh. (maximum members of clusters)	CPU (min)
Christofides69-50×5	160	582.7	582.7	0.00	1	5(11)	2.11
Christofides69-75×10	140	886.3	886.3	0.00	1	10(8)	3.60

⁹ instances are available in http://sweet.ua.pt/~iscf143/_private/SergioBarretoHomePage.htm.

Christofides69-100×10	200	889.4	889.4	0.00	1	8(13)	5.98
Gaskell67-21×5	6000	432.7	427.7	-1.16	2	4(7)	0.12
Gaskell67-22×5	4500	587.9	591.5	+0.61	1	3(14)	0.91
Gaskell67-29×5	4500	512.1	522.4	+2.01	1	3(16)	0.82
Gaskell67-32×5	8000	570.5	567.2	-0.58	1	4(11)	1.21
Gaskell67-32×5	11000	510.9	504.3*	-1.29	1	3(13)	1.00
Gaskell67-36×5	250	470.7	469.2	-0.32	1	4(9)	1.43
Perl83-12×2	140	204	205.3	+0.64	1	2(6)	0.03
Perl83-55×15	120	1127.1	1183.1	+4.97	3	10(6)	2.91

5. Conclusion

In this paper, based on importance of logistic systems and supporting supply chain, a new method was presented for capacitated location-routing problem. The aim of CLRP is that while considering problem hypotheses, proper depots are established and the optimum tour of vehicles is formed. The evaluation criterion of this goal is costs of depot location and vehicle routes which must be minimized. The presented heuristic method of the paper uses minimum number of vehicles and depots to solve the problem which can be considered as the aim in such problems. The heuristic method is presented based on 4 phases: in phase 1 of the algorithm, the customers are clustered according to greedy search method. Phase 2 determines the place of establishing depots, according to gravity center of clusters. In phase 3, clusters of customers are allocated to opened depots according to distance and capacity of the depot. Finally, phase 4 using ACS, finds proper tour between depot and allocated clusters. The results showed that suggested method in comparison to other methods, is more efficient both for answers' quality and solution time. Answers of 4 standard problems improved, 4 ones did not change and 3 ones had worse answers. The consideration show that the reason of worsening these 3 answers is the represented method of clustering. In future researches, we can use more effective methods of customers clustering.

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