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Activity Scheduling for Cost-Time Investment Optimization in Project Management

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Abstract

The Cost-Time Profile is a tool that presents the combined impact of disbursements made during the execution of a project, and their timing. The Cost-Time Profile chart presents the accumulated cost at any given time, and the area under the curve is the Cost-Time Investment. This Cost-Time Investment has a quantifiable impact on the working capital the company uses, and it is also useful for the determination of the Direct Cost of a product. This paper aims to develop a model to create the schedule of activities that minimizes the Cost-Time Investment for a project.

1. Introduction

It is universally recognized that money has value, and it also has value related to its position in time. If somebody owes money, they would prefer to pay their loan later rather than earlier. If some revenue is expected, it would be much better to receive it now instead of next week. This principle is more than applicable in this era of Just-In-Time, when activities are being performed when needed and not before. This paper briefly presents the Cost-Time Profile (CTP), which is a simple, graphical tool to consider both dimensions of cost (dollar amount and timing), and the resulting Cost-Time Investment (CTI - the area under the CTP curve, which represents the combination of how much money and for how much time it has been invested on a project), to finally propose a Mixed-Integer Program to schedule the activities of a project in such a way that the resulting CTI is minimal. In the following subsections we will present how to get there, starting from the concept and construction of a Cost-Time Profile and ending with the MIP model to create an optimal schedule that minimizes the CTI.

2. Methodology

2.1. What is a Cost-Time Profile (CTP)?

A Cost-Time Profile is a graph that depicts the Accumulated Costs that have been expended during the execution of a project at every time unit during the process. This way of presenting the information follows the use of resources through time, from the moment the execution begins until the company recovers those invested resources through the sale of the product. The area under the CTP is called the Cost-Time Investment (CTI), because it presents how much money has been tied up in the manufacturing process and for how long before being recovered through sales. The reader will recognize the term Investment because the CTI shares the two common components of any financial investment: Money and Time. **Figure 1** gives a simple illustration of a Cost-Time Profile.

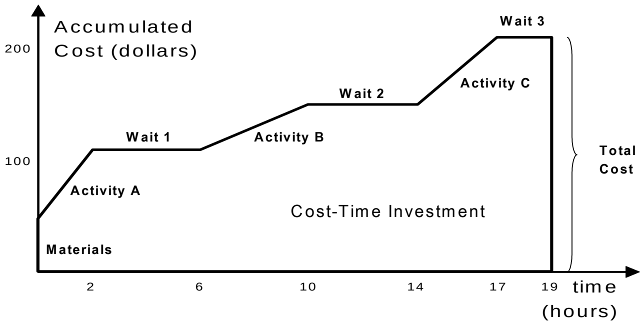

Figure 1. Example of a Cost-Time Profile

2.2. How do you build a CTP?

There are several important parts on a CTP.

– *Activities:* These are the parts that actively add cost. They are represented by positive-slope lines.

– *Materials:* Some activities require materials to be performed. The assumption is that the materials arrive right at the beginning of the activity and all at once, therefore they are represented by a vertical line (instantaneous accumulation of cost).

– *Waits:* These are moments when nothing that adds cost is actively happening. Waits are of interest mainly for manufacturing activities, because in projects there is always some cost happening (overhead?), even if you are not actively operating on the project. Since they do not add cost, they are represented by flat lines in the CTP curve (the accumulated cost remains constant during the wait).

– *Total Cost:* This is the height of the curve at the end of the project. It represents the total accumulated cost of the project.

– *Cost-Time Investment (CTI):* This is the area under the CTP curve, and it represents how much money and for how long has it been invested in the project. This is a measure of the utilization of working capital, and since it is capital you are using, this capital will undoubtedly have a cost that will be whatever rate the company has to pay for its working capital.

– *Working Capital Cost (WCC):* This is the cost the company has to pay for the use of capital. It can be expressed as in Equation 1.

$$WCC = CTI * \text{Cost of Capital Rate} \quad (1)$$

To build a CTP, we need to know several things. First, we need to know *when* is each element of the CTP happening (activities, waits and materials releases). We also need to know *how much* does each of these elements cost. Putting the two previous points together we can determine *how much* money is being spent as cost *at every time* unit in the process. Finally, we tally the costs and build the CTP with the *accumulated cost* at every time unit, and present this information in graphical form. The area under the curve (obtained adding the accumulated cost at each time unit for all the time units) represents the Cost-Time Investment. In the following subsections, a brief discussion on how to complete these steps for the construction of the CTP will be presented.

2.3. Applicability of CTP to Project Management

CTP does not consider indirect costs because it is truly hard to assign these costs directly to activities or individual units of a product. However, in project management it is reasonable to assume that all resources, people and equipment that take part in the project are directly and completely assignable to the project. Therefore, CTP is especially applicable to projects and the full spectrum of costs is included in them.

2.4. What is the contribution of each activity to the Cost-Time Investment? [3]

In order to create an optimal schedule of activities, it is necessary to characterize the contribution to the area under the CTP curve that each activity makes. If we consider that materials are always released at the beginning of an activity, we can see that Figure 2 presents the typical areas that we would see as a result of a given activity.

For Figure 2 we have that:

- Y = End date of the project
- X_i = Time when activity i finishes
- T_i = Duration of activity i
- MAT_i =Materials required to perform activity i
- CR_i = Cost rate of activity i (cost per time unit)

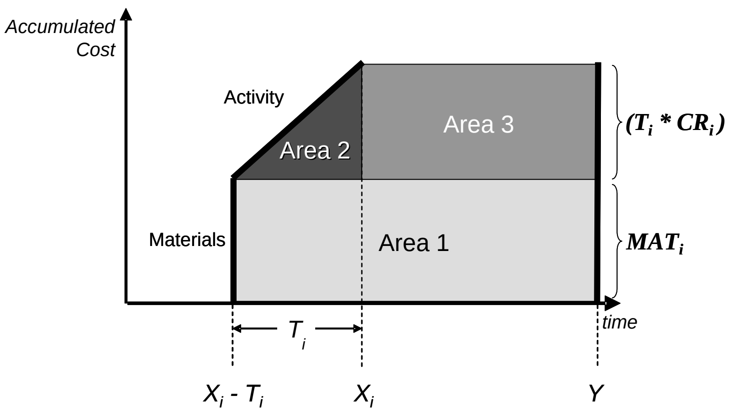

Figure 2: Areas that an activity contributes to the CTI.

- *Area 1* occurs when we incorporate the expenditure in materials, because they remain spent until the end of the project (Y). Area 1 can be calculated with Equation 2.

Area 1i = [Y – (Xi – Ti)] * MATi (2)

- *Area 2* occurs when the activity is actively happening. See Equation 3.

$$Area\ 2_i = \frac{(CR_i * T_i) * T_i}{2} = \frac{CR_i * T_i^2}{2} \quad (3)$$

- *Area 3* occurs from the end of the activity until the end of the project. This is the cost added by the activity projected until the end date of the project. Area 3 can be found using Equation 4.

Area 3i = (CRi * Ti) * (Yi – Xi) (4)

Adding Equations 2, 3 and 4 and organizing some terms we obtain the total contribution that each activity makes to the Cost-Time Investment. See Equation 5.

$$Area - Contribution_i = (Y - X_i)(CR_i * T_i + MAT_i) + T_i * MAT_i + \frac{CR_i * T_i^2}{2} \quad (5)$$

2.5. Traditional LP model for Project Management

The traditional LP model for project management (found in Operations Research or Management Science textbooks) considers precedence relationships between activities, and its objective function is to minimize the total completion time of a project. Figure 3 presents the text of such a model.

These models have two problems for its use with Cost-Time profiles:

Their objective function minimizes completion time of the project, not total Cost-Time Investment

They consider precedence constraints, but they do not recognize *resource dependencies* that happen when we have limited resources that are shared by different activities. These shared resources sometimes force that two activities that could be performed simultaneously will have to be scheduled one after the other.

In subsections 2.6 and 2.7 the solutions to these two problems will be presented.

MODEL 1		
Set:	Activities {A, B, C,…} (n activities).	
Indices:	*i, j* on Activities.	
Parameters:	T_i	Execution time for Activity *i*
	$PD_{i,j}$	1 if Activity *j* is an immediate predecessor of Activity *i* 0 otherwise
Variables:	X_i	Completion time for Activity *i*.
	Y	Completion time for the whole project
Objective Function:	Minimize the completion time for the project $Min\ Z = Y$	
Constraints:	Completion time for the project must be greater than or equal to the completion time for any activity $Y \geq X_i$; *for all i.* Any activity must only begin after its immediate predecessors have been completed. $X_i \geq X_j + PD_{i,j} * T_j$; *for all* $i \neq j$ (This constraint is only enforced when the $PD_{i,j}$ is 1, that is, when there exists a relationship of precedence between i and j).	

Figure 3: Traditional Project Management LP model.

2.6. Changing the Objective Function

If the objective of the model is the minimization of the Cost-Time Investment, then the objective function must be to minimize the sum of the CTI contributions of all the activities. That is, the objective function should be based in Equation 5. Figure 4 presents this updated model.

MODEL 2		
Set:	Activities {A, B, C,…} (n activities).	
Indices:	*i, j* on Activities.	
Parameters:	T_i	Execution time for Activity *i*
	$PD_{i,j}$	1 if Activity *j* is an immediate predecessor of Activity *i*; 0 otherwise
	CR_i	Cost Rate of Activity i
	MAT_i	Materials released at the beginning of Activity i
	T_i	Duration of Activity i
Variables:	X_i	Completion time for Activity *i*.
	Y	Completion time for the whole project
Objective Function:	Minimize the sum of contributions to the total area by the activities	
	$\text{Min Z} = \sum_i \left[(Y - X_i)(CR_i * T_i + MAT_i) + T_i * MAT_i + \frac{CR_i * T_i^2}{2} \right]$	
Constraints:	$Y \geq X_i$; *for all i.*	
	$X_i \geq X_j + PD_{i,j} * T_j$; *for all* $i \neq j$	

Figure 4: Project Management Model with CTI minimization as the objective function.

2.7. Shared resources constraints [3]

Projects have precedence constraints between activities. You should have the structure of a building before you can put up its walls. In this case, it is clear which activity needs to be completed before the dependent one starts. However, there are cases where activities do not have precedence relationships (you can landscape the front yard and the back yard in any sequence), but they cannot be performed simultaneously because they use the same resources (we only have one lawn mower, for example). When these resources are shared, it is not immediately evident which activity should be completed before the other. Therefore, a couple of new constraints and binary variables will be introduced to consider this type of situation.

Construction of a logical statement using binary variables: In the case of shared resources we need to tell the MIP that the activities that share a resource (let's call them *a* and *b*) will happen one before the other, but NEVER at the same time. So, it must be true that:

$Xa \geq Xb + Ta$ (if a starts after b is complete) XOR $Xb \geq Xa + Tb$ (if b starts after a is complete) (6)

(XOR is the eXclusive OR, meaning that one proposition or the other should happen, but NEVER both).

This XOR can be achieved using a binary variable called $y1_{ij}$, and two other new elements appear:

- SR_{ij}: This is a parameter. Its value is 1 if activities i and j share a resource and 0 otherwise.
- *M*: M is a BIG number that will be used to enforce or relax a constraint according to the values of the $y1_{ij}$

So, we will present the two new constraints and then offer some comments about the way they would work.

$X_i \geq T_i + (X_j * SR_{i,j}) - (M * y1_{i,j} * SR_{i,j})$ (If *j* needs to happen before *i*) (7.1)

$Xj \geq Tj + (Xi * SRj,i) - (M * y1j,i * SRj,i)$ (If i needs to happen before j) (7.2)

$y1i,j + y1j,i = 1$; for all $i \neq j$ (y1ij and y1ji are binary variables) (8)

Now, for the sake of argument, let's examine what happens in different situations with Equations 7.1 and 7.2.

Activities i and j do not share resources: When this is the case, the parameters SR_{ij} and SR_{ji} must both be zero, therefore Equations 7.1 and 7.2 transform in the following way

$Xi \geq Ti$ (7.1)

and

$Xj \geq Tj$ (7.2)

These two constraints do not add any trouble to the MIP, because it is naturally obvious that an activity's finishing time should be greater than or equal to its execution time.

Activities i and j share a resource: When this is the case, the parameters SR_{ij} and SR_{ji} must both be one. In this case we also need to consider the values of $y1_{ij}$ and $y1_{ji}$, and we know from equation 8 that one of them must be a one and the other one zero. Assuming that $y1_{ij} = 1$, and $y1_{ji} = 0$, then Equations 7.1 and 7.2 transform in the following way

$Xi \geq Ti + Xj - (M * y1i,j)$, that for all practical purposes will be $Xi \geq -M$ (7.1)

$Xj \geq Tj + Xi$ *(7.2)*

The real effect of using $y1_{ij}$ and $y1_{ji}$ will be that the constraint in Eq. 7.1 will be relaxed (not enforced), and equation 7.2 will be held, therefore the net result will be that activity j will happen after activity i. Had the $y1_{ij}$ and $y1_{ji}$ been the other way around ($y1_{ij} = 0$, and $y1_{ji} = 1$), activity i would happen after activity j. Integrating all these into the model, we obtain Model 3, presented in Figure 5

MODEL 3		
Set:	Activities {A, B, C,…} (n activities).	
Indices:	*i, j* on Activities.	
Parameters:	T_i	Execution time for Activity *i*
	$PD_{i,j}$	1 if Activity *j* is an immediate predecessor of Activity *i*; 0 otherwise
	$SR_{i,j}$	1 if Activities *i* and *j* share a Resource or an Operator. $SR_{i,j} = SR_{j,i}$ for all *i* and *j*. Also, $SR_{i,i} = 0$.
	CR_i	Cost Rate of Activity i
	MAT_i	Materials released at the beginning of Activity i
	T_i	Duration of Activity i
Variables:	X_i	Completion time for Activity *i*.
	Y	Completion time for the whole project
	$y1_{i,j}$	Binary Variables to assist in the logical *either-or* construction for Resource and Operator sharing
Objective Function:	Minimize the sum of contributions to the total area by the activities	

$$\text{Min Z} = \sum_i \left[(Y - X_i)(CR_i * T_i + MAT_i) + T_i * MAT_i + \frac{CR_i * T_i^2}{2} \right]$$

Constraints:

$Y \geq X_i$; *for all i.*

$X_i \geq X_j + PD_{i,j} * T_j$; *for all* $i \neq j$

$y1_{i,j} + y1_{j,i} = 1$; *for all* $i \neq j$

$X_i \geq T_i + (X_j * SR_{i,j}) - (M * y1_{i,j} * SR_{i,j})$; *for all* $i \neq j$

Figure 5: Model with CTI minimization as the objective function and shared resources

3. Conclusions and Future Research

The model presented in the previous section could be of great use for project oriented organizations, especially in those cases when the company works with its own resources before perceiving revenue (a real estate development company that builds government-subsidized housing, for example). In these cases the minimization of the completion is not as important an objective as it would be the minimization of the Cost-Time Investment (and the Cost-Time Profile would penalize lengthy projects anyway).

It is interesting to explore different objectives and constraints that might be included in Project Management models, since the specific characteristics of companies and their projects are so varied that specific models are always applicable to some companies.

Also, models with variable activity times (*a la* PERT) will be studied applying Monte Carlo simulation to explore the impact of specific activities on the resulting CTI and the differences in critical paths that could result.

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