A review of lot streaming in a flow shop environment with makespan criteria

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Abstract. In production scheduling, lot streaming is considered as the work setting in which the lots can be divided into smaller entities, called sublots. Dealing with a lot streaming problem means to widen the range of scenarios or typologies and an extension of the nomenclature and taxonomy is required. Aspects such as those related with the size of the sublots or the use of interleaving open a range of very interesting solutions to examine in different industrial environments. However, lot splitting makes it more complex to find optimal solutions. In this paper a comprehensive review of the literature will reveal the types of problems that have been addressed up to date, and with what degree of satisfaction have been resolved in the field of flow shop to minimize makespan (C_{max}) using lot streaming.

Keywords: flow shop, lot streaming, makespan, review, solution methods

1.1 Introduction

In the last sixty years thousands of papers have dealt with different scheduling issues related to flow shops configurations, and many others in its different variations. Most of these works have always been considered hypothesis, where jobs were not split. At the end of last century, and consolidated in the last decade, there arose a great interest in considering scenarios where the lots could be divided, that is what we call lot streaming. It seems clear that if it is possible, lot streaming minimize C_{max}. However, the difficulty in the resolution with this approach has, so far, prevented it can be considered a consolidated approach.

In the following section notation and structure of the problem will be presented, section 1.3 will review the two-machine cases, that are the basis to understand

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different approaches and to address more complex problems, such as those reviewed in section 1.4. And finally, section 1.5 discusses the techniques used to obtain the different solutions.

1.2 Notation

This paper is focus on flow shop problems where the number of stages and machines are the same; no multiple resources are available in any stage. All the reviewed flow shop lot streaming (FSLS) papers are presented on tables. These tables follow a modified notation of one previously published (Sarin & Jaiprakash, 2007): *{No. of machines}/{no. of jobs}/{sublot type}/{idling}/{sublot sizes}/{setup, special features}*

As we only deal with flow shop problems, we only specify the number of machines on it (2, 3 or N). Number of jobs may be single job (1) or multiple jobs (N). Sublot types may refer to equal (E), consistent (C) and variable (V). Intermittent idling (II) or no-idling (NI) will be also specified. Real numbers will be expressed in continuous values (CV) and integer sublots in discrete values (DV). For setup times, if no setup time is considered (No-ST), if it is considered (ST) or if it is sequence dependent (SDST). Special features include conditions such as no-wait condition (No-wait), when it is considered removal times (RemT) or transportation times (TransT) or even when interleaving is allowed (Interleaving). Makespan is considered implicitly in all cases reviewed.

1.3 Lot Streaming in two-stage flow shop

The 2/*/E problem, with one or n jobs, it could be regarded as a simple sequence problem of equal sublots, using Johnson's rule (Johnson, 1954) to find the optimal sequences in the two-machine. As it may be observed on Table 1.1, only three problems have been founded. A single job problem with discrete values but not using setup times (Sen, Topaloglu, & Benli, 1998). Other paper proposed an n job problem with continuous values (Vickson & Alfredsson, 1992). Further analytical research was performed over the previous paper and sublot-attached setup times were incorporated into the model (Baker, 1995). Other authors considered setup times on the problem (Cetinkaya & Kayaligil, 1992; Kalir & Sarin, 2003).

For the 2/1/C using consistent sublots, the objective is to simply determine the optimal sublot sizes for all the machines. First paper on the matter with continuous values indicated when it was convenient the use of them (Potts & Baker, 1989). Later on, different forms of the problem existing in the literature were reviewed and some important structural insights were generalized using both, continuous and discrete values (Trietsch & Baker, 1993). Years later, a paper was presented

for determining both, number of sublots and sublot sizes for a single job problem, and also for the *n* job one, considering setup times and a no-wait flowshop (Sriskandarajah & Wagneur, 1999). Previously, an analytical solution was provided using discrete values, to the problem when no setup times were considered (Sen et al., 1998). Other authors used a network representation to analyze the structure of the optimal sublot allocation (Chen & Steiner, 1999). They proposed an efficient solution method based on the structural properties giving discrete results.

Table 1.1 Papers of two-stage flow shop

Problem	Author(s)	Problem	Author(s)
2/1/E/II/DV/{No-ST}	Sen 1998	2/N/C/II/CV/{No-ST}	Potts 1989
2/N/E/II/CV/{No-ST}	Vickson 1992	2/N/C/II/CV/{ST,RemT}	Cetinkaya 1994
2/N/E/II/CV/{ST}	Cetinkaya 1992, Baker 1995, Kalir 2003	2/N/C/II/CV/{ST}	Vickson 1995
		2/N/C/II/CV/{ST,No-wait}	Sriskandarajah 1999
2/1/C/NI/CV/{No-ST}	Potts 1989, Trietsch 1993	2/N/C/II/DV/{ST}	Vickson 1995, Ganapathy 2004, Marimuthu et al. 2004, 2005
2/1/C/II/CV/{ST, No-wait}	Sriskandarajah 1999	2/N/C/II/DV/{ST,RemT}	Cetinkaya 1994
2/1/C/NI/DV/{No-ST}	Trietsch 1993	2/N/C/II/DV/{ST,No-wait}	Sriskandarajah 1999
2/1/C/II/DV/{No-ST}	Sen 1998, Chen 1999	2/N/C/II/DV/{ST,TransT, Interleaving}	Cetinkaya 2006
2/1/C/II/DV/{ST, No-Wait}	Sriskandarajah 1999	2/1/V/II/CV/{No-ST}	Sen 1998

For the 2/N/C/II/CV, we need to simultaneously obtain the best job sequence and the optimal sublot allocation (sublot starting and completion times). All the papers allowed intermittent idling. It was showed that it is not possible to solve the *n*-job problem simply by applying lot streaming individually to the single-job problem (Potts & Baker, 1989). Several papers independently show that this problem it is decomposed into an easily identifiable sequence of single job problems, using continuous values, even with setup times (Vickson, 1995) and transfer times (Cetinkaya, 1994). Other authors have widely tackled the same problem using discrete values (2/N/C/II/DV) considering setup times (Ganapathy, Marimuthu, & Ponnambalam, 2004; Marimuthu & Ponnambalam, 2005; Marimuthu, Ponnambalam, & Suresh, 2004). Sublot attached and detached setup times were also considered (Vickson, 1995). It was presented some closed form solutions for continuous sublots and a fast polynominally bounded search algorithm for discrete sublots. Other papers proposed the use of removal times (Cetinkaya, 1994), of no-

wait condition (Sriskandarajah & Wagneur, 1999) or even allowing interleaving (Cetinkaya, 2006).

Using variable sublots in a 2/*/V problem, only a paper was founded. Due to the complexity that involves variable sublots, it calculated continuous values and it did not consider setup times (Sen et al., 1998).

1.4 Lot streaming in m-stage flow shop

For the problems with more than two-machine, papers published on the topic are displayed on the Table 1.2. For the 3/N/E problem, Johnson's rule was modified to obtain the optimal solution with unit-size sublots and continuous values (Vickson & Alfredsson, 1992). Equal-sized sublots are popular in practice. These were first studied in an m/1/E problem, where setup times were considered (Truscott, 1985). Later on a bottleneck minimal idleness heuristic (BMI) was developed to generate solutions that were very close to the optimum (Kalir & Sarin, 2001). For the m/N/E problem, the BMI model was extended to *n* jobs but it did not consider set-up times on it (Kalir & Sarin, 2001b). Other paper used integer programming to determine optimum sublot sizes while enumerating the number of sublots for an *n* jobs problem using discrete values (Huq, Cutright, & Martin, 2004). Other re-searchers presented five methods including a tabu search (TS), simulated anneal-ing (SA), hybrid genetic algorithm (HGA), ant colony optimization (ACO) and threshold accepting (TA) algorithms involving attached setup times (Marimuthu, Ponnambalam, & Jawahar, 2007, 2008, 2009). Idling and no-idling condition was added to the problem (Pan, Wang, Gao, & Li, 2011).

Linear and integer programming formulations were presented to determine optimal sublot sizes for one job on a 3-machine flow shop (3/1/C) using both, continuous and discrete values with consistent sublots (Trietsch & Baker, 1993). Years later, no-wait condition was added to the problem (Wagneur, 2001). Other authors extended to the case containing detached (Chen & Steiner, 1997a) and at-tached (Chen & Steiner, 1998) setup times. For the case of m/1/C/CV, it was ex-tended a previous work (Sriskandarajah & Wagneur, 1999) and it was used genet-ic algorithm (GA) to solve problems in which fixed and variable numbers of sublots for each product were included (Kumar, Bagchi, & Sriskandarajah, 2000). For the m/1/C/DV, Glass and Potts proved that only dominant machines may ap-pear on a critical path (Glass & Potts, 1998). Years later, a heuristic using discrete sublot sizes and no setup times was proposed (Edis & Ornek, 2009). Most of the papers used different methods to convert continuous into discrete sublot sizes (Chen & Steiner, 1997b, 2003; Glass & Herer, 2006). Multi-objective lot stream-ing problem (minimizing makespan and mean flow time simultaneously) was in-vestigated (Bukchin & Masin, 2004). They also considered setup times such as (Kumar et al., 2000), who considered no-wait condition like (Chen & Steiner, 2003).

Table 1.2 Paper of more than 2-stage flow shop

Problem	Author(s)	Problem	Author(s)
3/N/E/II/CV/{No-ST}	Vickson 1992	m/1/C/II/DV/{No-ST}	Glass 2006
m/1/E/NI/CV/{ST}	Truscott 1985	m/1/C/II/DV/{No-ST,No-Wait}	Chen 2003
m/1/E/II/CV/{ST}	Kalir et al. 2001	m/1/C/II/DV/{ST}	Buckhin 2004
		m/1/C/II/DV/{ST,No-wait}	Kumar et al. 2000
m/N/E/II/CV/{No-ST}	Kalir 2001	m/N/C/II/CV/{ST,No-wait}	Kumar et al. 2000
m/N/E/II/DV/{ST}	Huq 2004, Marimuthu et al. 2007, 2008, 2009	m/N/C/II/CV/{ST,Interleaving}	Bukchin 2010
		m/N/C/II/DV/{ST,No-wait}	Kumar et al.2000, Hall 2003, Kim 2009
m/N/E/{II,NI}/DV/{ST}	Pan et al. 2011	m/N/C/II/DV/{No-ST, Interleaving}	Feldmann 2008
		m/N/C/II/DV/{ST,Interleaving}	Martin 2009
3/1/C/{NI,II}/CV/{No-ST}	Trietsch 1993	m/N/C/II/DV/{SDST}	Pan et al. 2010a, 2010b
3/1/C/II/CV/{No-ST,No-wait}	Wagneur 2001	m/N/C/{II,NI}/DV/{SDST}	Pan & Ruiz 2011
3/1/C/{NI,II}/DV/{ST}	Chen 1997b, 1998	3/1/V/{NI,II}/CV/{No-ST}	Trietsch 1993
m/1/C/II/CV/{ST,No-wait}	Kumar et al. 2000	3/1/V/{NI,II}/DV/{No-ST}	Trietsch 1993
m/1/C/{NI,II}/DV/{No-ST}	Glass 1998, Edis 2009	m/1/V/II/DV/{No-ST,No-Wait}	Liu 2003
m/1/C/II/DV/{No-ST}	Chen 1997	m/1/V/NI/DV/{ST,Transp}	Chiu 2004
		m/N/V/II/DV/{ST}	Defersha 2010

For the problem of m/N/C/CV an heuristic and the use of GA for sequencing the products and for determining the number of sublots were proposed (Kumar et al., 2000). Bukchin extended his previous work in m/1/C to n jobs, but this time allowing interleaving (Bukchin, Masin, & Kirshner, 2010). Many researchers studied the no-wait FSLS problems not allowing interleaving but integer sizes (m/N/C/DV) were assumed (Hall, Laporte, Selvarajah, & Sriskandarajah, 2003; Kim & Jeong, 2009; Kumar et al., 2000). Other authors allowed the use of interleaving among different jobs (such as Bukchin but using discrete values), not considering setup times (Feldmann & Biskup, 2008) or considering them (Martin, 2009). Other authors focused on sequence dependent setup times (Pan, Duan, Liang, Gao, & Li, 2010a; Pan, Tasgetiren, Suganthan, & Liang, 2010b) and included no-idling condition (Pan & Ruiz, 2012).

For the 3/1/V problem, no setup times were considered in both cases, with consistent and discrete values (Trietsch & Baker, 1993). A heuristic method was proposed for the m/1/V problem with no setup times and no-wait condition (Liu,

2003). Later on, other paper considered transportation and setup times (Chiu, Chang, & Lee, 2004).

For the m/N/V, only one paper has been founded in which it was considered setup times (Defersha & Chen, 2010).

1.5 Method used in flow shop lot streaming

In the two previous sections, efforts have focused on analyzing the types of problems addressed and the satisfaction achieved with the proposed solutions. This section introduces a classification of techniques that have been used in the papers reviewed and a brief analysis of them.
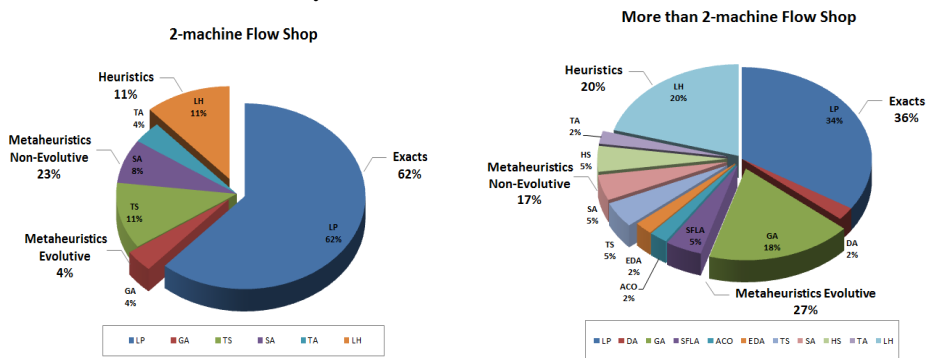

Fig. 1.1 Methods used for two-machine and m-machine flow shop.

The methods used have been classified in exact and approximate, being the last type divided in meta-heuristics (Evolutionary and Non-evolutionary) and heuristics. As it is shown in Figure 1.1, for the simple case of two-machine, exact methods dominate proposed solutions. From the 62% of the exact solutions proposed, most of them focused on the approach of a MILP model which is then analytically developed hypotheses allowing, in some cases in other dimensions theorems for minimizing C_{max}. 11% are heuristics, usually developed from the MILP model analysis, and 23% are traditional meta-heuristics, evolutionary methods only represent 4%. In Figure 1.1 also shows the distribution of techniques employed in the case of more than two machines. As you can see the use of exact methods is reduced to 36%, although they have been used to simplified cases (few jobs). The evolutionary methods achieve a significant 27%, while non-evolutionary meta-heuristic and heuristics techniques make a similar contribution ($\approx$20% both).

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